System Inversion from the Perspective of Inverse Model Feedforward with Application to a Nonminimum-Phase Benchmark System

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Inverse model feedforward enables high performance in tracking control applications. The aim of this paper is to compare several model inversion techniques for nonminimum-phase systems. The techniques are applied to a nonminimum-phase benchmark system and evaluated in various aspects, including finite-time versus infinite-time design, amount of preview, and tracking performance. The results show limitations of traditional techniques and the potential of recently developed techniques. The presented inversion techniques enable high performance for nonminimum-phase systems in tracking control applications.

Keywords: Model inversion, Nonminimum phase, Inverse model feedforward

1. Motivation

System inversion is essential in tracking control applications, including inverse model feedforward and learning control. The model to be inverted includes the open-loop system in inverse model feedforward, the closed-loop process sensitivity in iterative learning control (ILC), or the closed-loop complementary sensitivity in repetitive control.

System inversion of nonminimum-phase (NMP) systems is highly challenging, e.g., direct inversion may yield an unbounded response. The aim of this paper is to investigate, compare, and develop inversion techniques for the purpose of inverse model feedforward.

2. Challenges in inversion

In this paper, the general block diagram in Fig. 1 is considered. Throughout, it is assumed that discrete-time system H is linear time-invariant (LTI), nonminimum phase, and has state-space realization (A, B, C, D). If D is invertible, an immediate solution to obtain $e = 0$ is to select $F = H^{-1}$, where

$$H^{-1} \stackrel{s}{=} \left[\begin{array}{c|c} A - BD^{-1}C & BD^{-1} \\ \hline -D^{-1}C & D^{-1} \end{array} \right]. \qquad (1)$$

However, since H is nonminimum-phase, the direct use of (1) yields unbounded u when solved forward in time.

Given Fig. 1, the objective is to find bounded u such that $e \approx 0$. Typically, $F \approx H^{-1}$ is pursued [1], regardless of the control objective. In this paper, the control objective of inverse model feedforward is considered, in particular the design of F which minimizes $\|e\|_2$ with $e = (I - HF)r$, see also Fig. 1. The key aspect is the use of noncausal F, which is generally allowed in feedforward control.

3. Inversion techniques

In this paper, the inversion techniques in Table 1 are considered. The approximate inverse techniques NPZ-Ignore,

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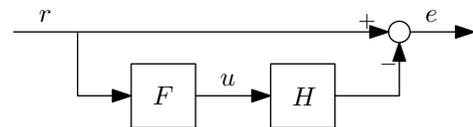* Control Systems Technology, Eindhoven University of Technology, Eindhoven, The Netherlands

Fig. 1.   Given H, the aim is to design F such that $\|e\|_2$ is minimized, while u remains bounded.

Table 1.   Overview of the inversion techniques and their characteristics.

Technique	Design	Preview	Dimensions	Aim
NPZ-Ignore [2]	Infinite	Finite	scalar	H^{-1} approx.
ZPETC [4]	Infinite	Finite	scalar	H^{-1} approx.
ZMETC [3]	Infinite	Finite	scalar	H^{-1} approx.
Stable inversion [5]	Infinite	Infinite	Square	H^{-1} exact
Norm-optimal feedforward [5]	Finite	Infinite	Non-square	$\min \|e\|_2$
$\mathcal{H}_\infty$-preview control [6]	Infinite	Finite	Non-square	$\min \|I - HF\|_\infty$
$\mathcal{H}_2$-preview control [6]	Infinite	Finite	Non-square	$\min \|e\|_2$

ZPETC, and ZMETC are based on $H(z)F(z)$ (e.g., zero phase) and do not address the control objective of minimizing $\|e\|_2$. Stable inversion, norm-optimal feedforward, and $\mathcal{H}_2$-preview control address the control objective in various manners. $\mathcal{H}_\infty$-preview control does not address the control goal, but is in fact useful for iterative learning control (ILC) [6].

4. Benchmark system

The inversion techniques are validated on the benchmark system shown in Fig. 2 with transfer $u \mapsto y$ given by

$$H(z) = \frac{-3 \times 10^{-8}(z + 0.9632)(z - 0.9447)(z - 1.1410)}{(z - 1)^2(z^2 - 1.9595z + 0.9632)}, \qquad (2)$$

with sample time $h = 0.001$ s. System H has one nonminimum-phase zero $z = 1.1410$, see also Fig. 2(b).

5. Application in inverse model feedforward

Application of the inversion techniques in Table 1 to the system in Fig. 2 yields the results in Fig. 3 and Fig. 4.

The inputs u are shown in Fig. 3. Fig. 3(a) shows that regular inversion (1) yields an unbounded input and that the inputs of NPZ-Ignore and ZPETC are not very well suited for
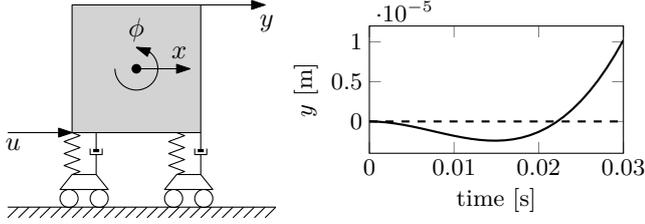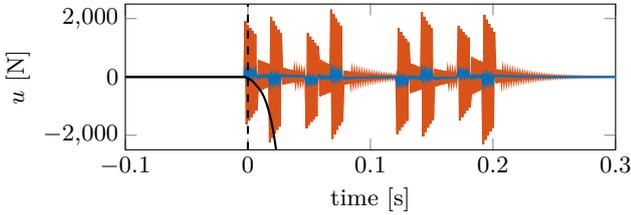
(a) The mass with input u and output y can translate in x and rotate in ϕ.

(b) The step response of H in (2) initially moves in opposite direction due to the NMP zero.
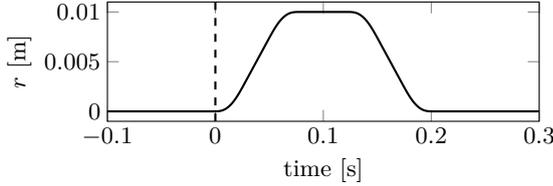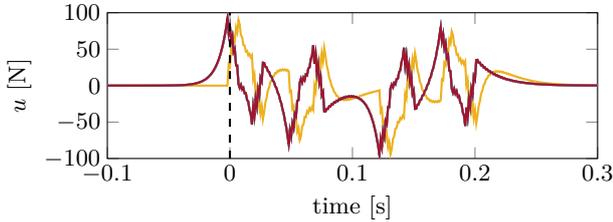
(c) Reference trajectory r includes pre-actuation and post-actuation time.

Fig. 2.   Benchmark system and reference trajectory.

(a) NPZ-Ignore (——), ZPETC (——), and regular inversion (——). Regular inversion (1) yields an unbounded input which is undesired.
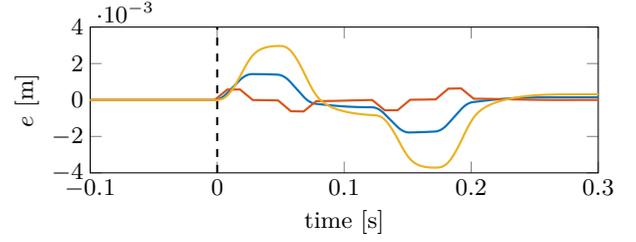
(b) ZMETC (——), stable inversion (——), norm-optimal feedforward (——), $\mathcal{H}_\infty$-preview control (——), and $\mathcal{H}_2$-preview control (——). All except ZMETC are overlapping.

Fig. 3.   The inversion techniques in (b) yield smaller input signals u than the inversion techniques in (a).

(a) NPZ-Ignore (——), ZPETC (——), and ZMETC (——).

(b) Stable inversion (——), norm-optimal feedforward (——), $\mathcal{H}_\infty$-preview control (——), and $\mathcal{H}_2$-preview control (——). Norm-optimal feedforward, stable inversion, and $\mathcal{H}_2$-preview control are overlapping.

Fig. 4.   The inversion techniques in (b) yield small error signals e, in contrast to the inversion techniques in (a).
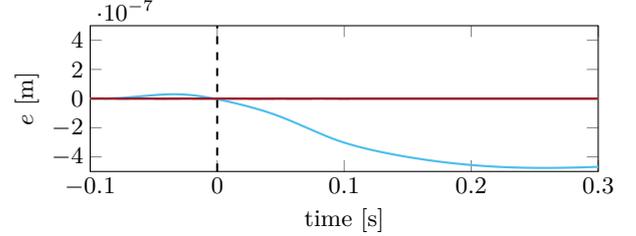
practical application, which is in line with [1, Sec. 6]. The corresponding error signals in Fig. 4 are also considerably large. Fig. 3(b) shows that the inputs of all approaches, except for ZMETC, are similar.

The error signals e are shown in Fig. 4. Fig. 4(a) shows a large error for the approximate inverse techniques. Fig. 4(b) shows that stable inversion, norm-optimal feedforward, and $\mathcal{H}_2$-preview control achieve the lowest $\|e\|_2$, which is to be expected based on their properties, see Table 1. $\mathcal{H}_\infty$-preview control achieves moderate performance since it aims at minimizing $\|I - HF\|_\infty$ rather than $\|e\|_2$.

The results confirm the analysis in section 3. In particular, NPZ-Ignore, ZPETC, ZMETC, and $\mathcal{H}_\infty$-preview control yield moderate performance since they do not address the control objective. In contrast, stable inversion, norm-optimal feedforward, and $\mathcal{H}_2$-preview control yield excellent performance since they address the control objective. The particu-

lar choice depends on aspects such as the amount of preview and the dimensions of the system.

6. Conclusion

System inversion is essential in inverse model feedforward. Direct inversion of a nonminimum-phase system yields an unbounded response. Bounded responses can be obtained through inversion techniques. In this paper, several inversion techniques are evaluated from the perspective of inverse model feedforward. From this perspective, stable inversion, norm-optimal feedforward, and $\mathcal{H}_2$-preview control are preferred since they explicitly address the control objective.

Future work focuses on inversion techniques for different control applications such as ILC and for different classes of systems, such as time-varying and non-square systems.

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