Improved state estimation by non-causal state observer

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State estimation is essential for tracking conditions which can not be directly measured by sensors or are too noisy. The aim of this paper is to present an approach to mitigate the phase delay without compromising the noise sensitivity, by using accessible future data. Such use of future data can be possible in cases like Iterative Learning Control, where full data of the previous trial is acquired beforehand. The effectiveness of the presented approach is verified through LTI motion systems with 1) plant model error and 2) input disturbance, successfully showing the state estimation improvement from both time and frequency domain. The proposed non-causal approach improves the trade-offs between the phase delay of the estimation and the noise resistance of the state observer.

Keywords: Iterative Learning Control, State Observer, Kalman Smoothing, Fixed-Interval Smoothing, Stable Inversion

1. Introduction

Moore's Law has been a driving force for the development of the industry, contributing for the rise of our standards of living [1]. This also means the demand of accuracy and speed of controlling machines are growing exponentially. To answer this problem, various solutions are considered. For example, using light weight and cheap non-metal materials in the machines to increase the productivity of the manufacturing [2]. However, in order to effectively implement such materials, consideration of the flexible mode is inevitable. Current methods using Feed-back (FB) and Feed-forward (FF) control are still powerful, but since FF is vulnerable to plant model error and FB is subject to fundamental performance limitations such as analytic and interpolation constraints, perfect tracking is far from realized.

Iterative Learning Control (ILC), which is a method that "learns" from previous trials, have started to catch attention. Due to ILC's batch-wise data processing, a non-causal filter, such as an stable inversion of a non-minimum phase system, is constructible. This means that against a repetitive task, an accurate and faster control than FB and FF can be accomplished [3]. One problem of ILC is that, making an accurate inversion of a system with a discretization zero close to the stability limit, causes an oscillation to the control input, resulting to a poor inter-sample behavior. To address this behavior, an approach focusing on controlling not only the output, but the entire state variables of the plant, called State-Tracking ILC has been proposed [4].

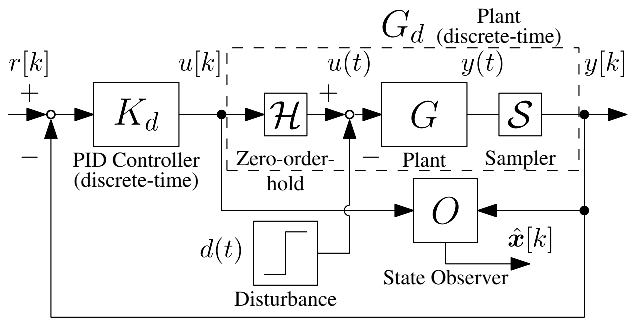

Fig. 1: The feedback system considered for state estimation.

Although State-Tracking ILC can achieve improved inter-sample behavior, the tracking performance depends on the estimated plant state by the implemented state observer, which has the fundamental trade-offs between the bandwidth and measurement noise sensitivity. The aim of this paper is to present an effectiveness of a non-causal state observer that utilizes the future signal from the past iteration by assuming repetitiveness. The non-causal state observer enables the improved state estimation accuracy without changing the bandwidth.

The outline of this paper is as follows. In Section 2, the control diagram is presented and the objective of state estimation is formulated. The core idea of the approach to utilize future data for the non-causal state observer is presented in Section 3. The strength of the approach are demonstrated by application to a simple Linear Time-Invariant (LTI) system simulation in Section 4. Section 5 concludes this paper.

2. Problem Formulation

In this section, the state estimation problem is formulated. A tracking control configuration shown in Fig.1 is considered to evaluate the state estimation error of state observers. With G being a continuous-time LTI SISO system, $\mathcal{S}$ being a sampler, and $\mathcal{H}$ being a zero-order hold, K_d is a digital controller.

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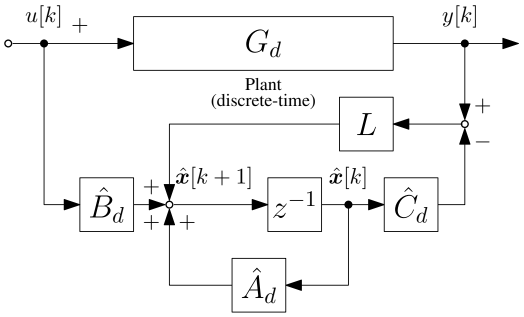


Fig. 2: The most basic full-order state observer.

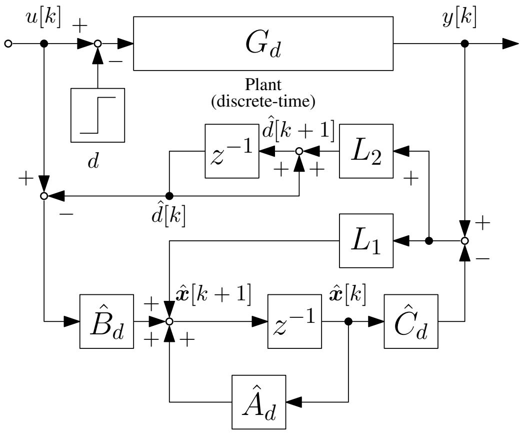


Fig. 3: An augmented full-order state observer estimating input disturbance.

r, u, y, and d each represent the reference trajectory, control input, output, and input disturbance, respectively.

When $d = 0$ and G is given by the minimal realization

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{B}_c u(t) \tag{1a}$$

$$y(t) = \boldsymbol{C}_c\boldsymbol{x}(t) \tag{1b}$$

a full-order state observer (Fig.2) can be constructed at sampling time δ as following

$$\hat{\boldsymbol{x}}[k+1] = \boldsymbol{A}_d\boldsymbol{x}[k] + \boldsymbol{B}_d u[k] + \boldsymbol{L}(y[k] - \hat{y}[k]) \tag{2a}$$

$$\hat{y}[k] = \boldsymbol{C}_d\boldsymbol{x}[k] \tag{2b}$$

with

$$\boldsymbol{A}_d = e^{\boldsymbol{A}_c\delta}, \quad \boldsymbol{B}_d = \int_0^{\delta} e^{\boldsymbol{A}_c\tau}\boldsymbol{B}_c d\tau, \quad \boldsymbol{C}_d = \boldsymbol{C}_c. \tag{2c}$$

$\boldsymbol{x}$, $\hat{\boldsymbol{x}}$, $\hat{y}$, and $\boldsymbol{L}$ denote state variable, state variable estimation, output estimation, and an observer gain matrix, respectively. $\boldsymbol{L}$ can be determined through a Discrete-time Algebraic Riccati Equation [5] for minimizing the cost function $J = \int_0^{\infty}(\boldsymbol{x}(t)^\top\boldsymbol{Q}\boldsymbol{x}(t) + u(t)^\top Ru(t))dt$ with $\boldsymbol{Q}$ and R being a weight decided by the designer.

When an input disturbance d is present, assuming that d is a step disturbance (i.e. $d[k+1] = d[k]$), an augmented full-order state observer with a disturbance estimation (Fig.3) can be constructed at sampling time δ as following

$$\begin{bmatrix}\hat{\boldsymbol{x}}[k+1]\\ \hat{d}[k+1]\end{bmatrix} = \begin{bmatrix}\boldsymbol{A}_d & -\boldsymbol{B}_d\\ \boldsymbol{O} & 1\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{x}}[k]\\ \hat{d}[k]\end{bmatrix} + \begin{bmatrix}\boldsymbol{B}_d\\ 0\end{bmatrix}u[k] + \begin{bmatrix}\boldsymbol{L}_1\\ L_2\end{bmatrix}(y[k] - \hat{y}[k]) \tag{3a}$$

$$\hat{y}[k] = \boldsymbol{C}_d\boldsymbol{x}[k] \tag{3b}$$

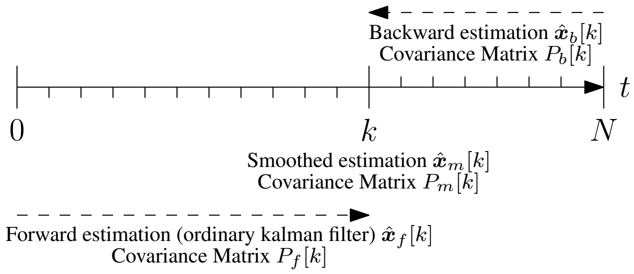
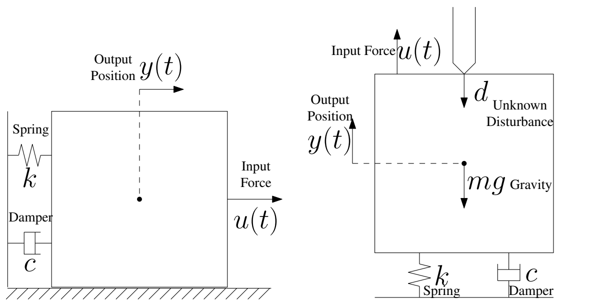


Fig. 4: Basic concept of Fixed-Interval Smoothing. By combining $\hat{\boldsymbol{x}}_f$ and $\hat{\boldsymbol{x}}_b$, a smoothed estimate $\hat{\boldsymbol{x}}_m$ is achieved.



(a) Plant with no disturbances. (b) Plant with a disturbance.

Fig. 5: Plant models used for the simulation.

with $\hat{d}$ being the input disturbance estimation. Observer gain $\begin{bmatrix}\boldsymbol{L}_1 & L_2\end{bmatrix}^\top$ can be determined through a Discrete-time Algebraic Riccati Equation for minimizing the cost function $J = \int_0^{\infty}([\boldsymbol{x}(t)^\top\ d(t)]\boldsymbol{Q}[\boldsymbol{x}(t)^\top\ d(t)]^\top + u(t)^\top Ru(t))dt$ with $\boldsymbol{Q}$ and R being a weight decided by the designer.

The aim of this paper is to investigate whether the proposed non-causal state observer can yield a better estimation $\hat{\boldsymbol{x}}$, $\hat{d}$ than the conventional observer.

3. Conceptual Idea

In this section, the conceptual idea of the non-causal state observer is presented.

The non-causal state observer is based on a Kalman Smoothing method called Fixed-Interval Smoothing. Fixed-Interval Smoothing is a state estimation which can be made when all of the input data is provided beforehand [6][7]. The basic concept of Fixed-Interval Smooting is shown in Fig.4. By combining the forward estimation $\hat{\boldsymbol{x}}_f$ and backward estimation $\hat{\boldsymbol{x}}_b$, a smoothed estimation $\hat{\boldsymbol{x}}_m$ is obtained. The smoothing is done by following Eq.4.

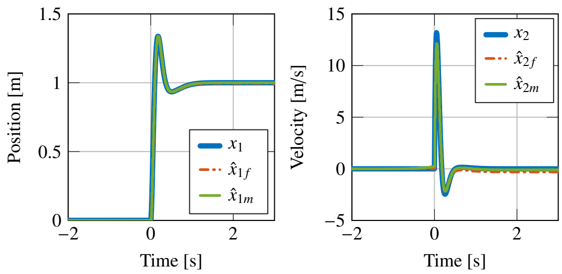
$$\hat{\boldsymbol{x}}_m = \boldsymbol{\alpha}\hat{\boldsymbol{x}}_f + (\boldsymbol{I} - \boldsymbol{\alpha})\hat{\boldsymbol{x}}_b \tag{4a}$$

$$\boldsymbol{P}_m = \left[(\boldsymbol{P}_f)^{-1} + (\boldsymbol{P}_b)^{-1}\right]^{-1} \tag{4b}$$

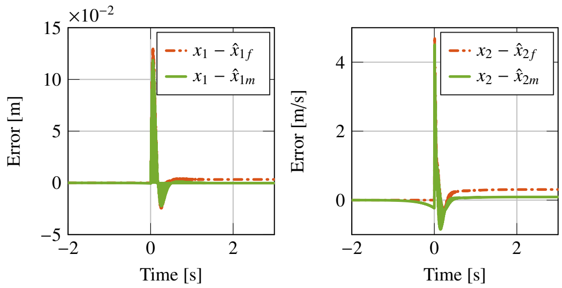
$$\boldsymbol{\alpha} = \boldsymbol{P}_b(\boldsymbol{P}_f + \boldsymbol{P}_b)^{-1} \tag{4c}$$

$\boldsymbol{P}_f$, $\boldsymbol{P}_b$, and $\boldsymbol{P}_m$ are the covariance matrix of the forward, backward, and smoothed estimation, respectively. Further information of Fixed-Interval Smoothing can be found in literature [6][7].

One problem with Fixed-Interval Smoothing is that in Linear Time-Variant systems, backward estimation $\hat{\boldsymbol{x}}_b$ is unreliable when k is close to N. Therefore, in this paper we apply an approach used in stable inversion [8] to obtain a more reliable $\hat{\boldsymbol{x}}_b$.



(a) State estimation result.



(b) State estimation error.

Fig. 6: Simulation results for Case 1. True state (▬), state estimation using conventional observers (-·), and state estimation using proposed non-causal observers (—) are shown. Results from (b) show that steady state estimation error can be mitigated by the proposed method.

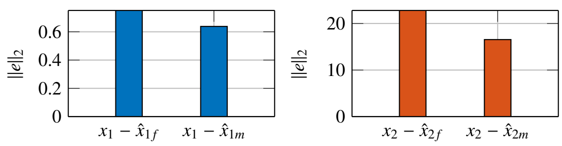
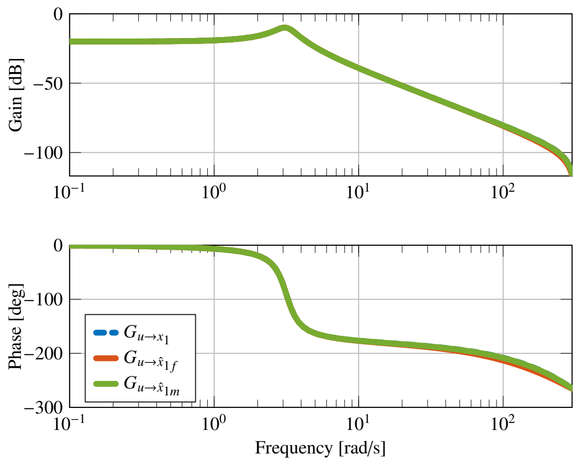
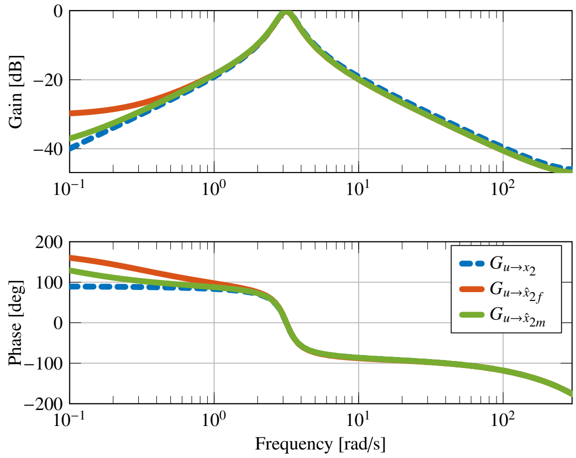


Fig. 7: Euclidean Norm of State Estimation Error for Case1. For both x_1 (▬) and x_2 (▬), state estimation by the non-causal observer performs with a lower state estimation error.



(a) Frequency characteristic of u (input) $\rightarrow \hat{\boldsymbol{x}}_1$ (position estimation).



(b) Frequency characteristic of u (input) $\rightarrow \hat{\boldsymbol{x}}_2$ (velocity estimation).

Fig. 8: Frequency characteristics for Case 1. Results show that non-causal observers (▬) achieve a closer characteristic to the true state (▪ ▪) than the conventional observer (▬).

Stable inversion is one of the methods used to obtain an inversion of a proper system H, even when it contains unstable zeros. Other approximate inversion methods such as NPZ-Ignore [9], ZPETC [10], ZMETC [11] are known as well, but one important characteristic of stable inversion is that, with an assumption of an infinite preactuation time, it has zero magnitude and zero phase error. This preactuation occurs due to calculating the unstable poles of H^{-1} backwards in time. We will be applying this calculation for our non-causal state observer, see [8] for further details.

The non-causal state observer is constructed as following. For forward estimation $\hat{\boldsymbol{x}}_f$, we use aposteriori estimation converted from ordinary state observer's apriori estimation. As explained in Section 2, ordinary state observers are formed by deciding a weight $\boldsymbol{Q}$ and R, yielding an observer gain $\boldsymbol{L}$ calculated from a unique stabilizing solution of the Discrete-time Algebraic Riccati Equation. The backward estimation $\hat{\boldsymbol{x}}_b$, is provided below. Using the same $\boldsymbol{Q}$ and R, an unique anti-stabilizing solution of the Discrete-time Algebraic Riccati Equation can yield an observer gain $\boldsymbol{L}'$. Directly using this $\boldsymbol{L}'$ will end up being unstable. However, using the backward calculating techniques in stable inversion, we will be able to obtain a stable state estimation with a preacuation. We will combine $\hat{\boldsymbol{x}}_f$ with $\hat{\boldsymbol{x}}_b$ to construct a non-causal state observer with a smoothed estimation $\hat{\boldsymbol{x}}_m$.

4. Simulation Results

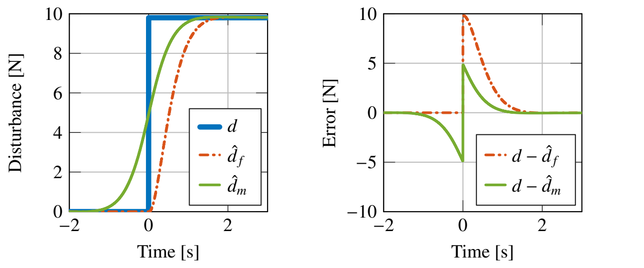
4.1 Simulation Plant and Simulation Condition The simplified models of the simulation plants are shown in Fig.5. The linear motor provides actuation force (input u) and moves the plant towards a predefined position trajectory. The position of the plant (output or control target y) is measured by a linear encoder.

In this paper we considered two situations.

First, we considered the most basic model without any disturbance. However, if we use complete information of the plant, the state observer will perfectly track the state variables. Therefore, we assumed a 10% gain error for the nominal plant model and constructed the state observer.

Second, we considered a model with an unknown input disturbance. In this case, we assumed a zero-order disturbance and constructed a state observer based on the assumption. For simplicity, we did not assume any model error for the nominal plant model.

For both cases we set, the mass of the object to $m = 1\,\mathrm{kg}$, spring constant to $k = 10\,\mathrm{N/m}$, and damper constant to $c = 1\,\mathrm{Ns/m}$. The simulation starts at $t_0 = 0\,\mathrm{s}$, and we as-



(a) State estimation result. (b) State estimation error.

Fig. 9: Simulation results for Case 2. Estimation result of the disturbance shows that the proposed non-causal observer (—) negates the phase error seen in the conventional observer (-·).

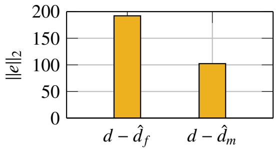


Fig. 10: Euclidean Norm of State Estimation Error for Case2. Decline of estimation error for disturbance (▬) can be seen using the non-causal observer.

sumed $y_0 = 0$ m, $u_0 = 0$ N. Controllers were controlled at a sampling time $T_s = 0.01$ s, and for case 2 a step disturbance $d = 9.8$ N starting at t_0 was assumed.

The transfer function from u to y, G can be calculated as

$$G(s) = \frac{1}{s^2 + s + 10} \tag{5}$$

the controllable canonical form being the following.

$$\left[\begin{array}{c|c} \boldsymbol{A}_c & \boldsymbol{B}_c \\ \hline \boldsymbol{C}_c & \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -10 & -1 & 1 \\ \hline 1 & 0 & \end{array}\right] \tag{6}$$

4.2 Case 1: Plant without an Input Disturbance As previously explained, we assumed a 10% gain error for this case, thus controllers and state observers will be constructed based on the nominal plant $G_n = 0.9G$.

We used a PID controller $K_d = 529.34\frac{z^2-1.935z+0.9366}{(z-1)(z-0.6736)}$, and for the state observer we set $\boldsymbol{Q} = \mathrm{diag}(1, 10)$, $R = 0.1$.

The result of the state estimation is shown in Fig.6a. Figure 6b shows the error of the state estimation, and from the result it can be seen that the non-causal state observer was able to negate the steady state error. The improvement can be observed from the euclidean norm of state estimation error shown in Fig.7 too.

Figure 8 is the frequency characteristic of $u \to \hat{\boldsymbol{x}}$. Comparing both results to the true frequency characteristic $u \to \boldsymbol{x}$, shows that non-causal state observers show a closer response than the conventional method.

4.3 Case 2: Plant with an Input Disturbance Having full knowledge of the plant G, we used a PID controller $K_d = 476.4\frac{z^2-1.935z+0.9366}{(z-1)(z-0.6736)}$, and for the state observer we set $\boldsymbol{Q} = \mathrm{diag}(1, 10, 100)$, $R = 0.1$.

The result of the state estimation is shown in Fig.9a. For simplicity, only the estimation of the disturbance is shown. Figure 9b shows the error of the input disturbance estimation,

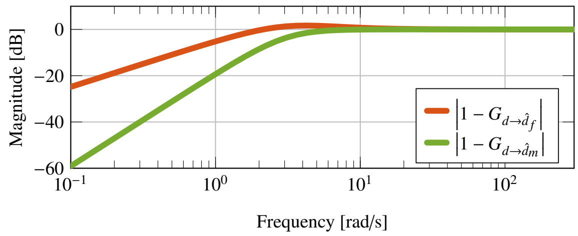


Fig. 11: The error in frequency characteristic of d (actual disturbance) $\to \hat{d}$ (estimated disturbance). In this figure the difference from the most ideal characteristic ($G_{d\to\hat{d}} = 1$) is shown. The proposed non-causal observer (▬) achieves a more desirable characteristic than the conventional observer (▬) especially in the low frequency range.

and from the result it can be seen that the non-causal state observer was able to cancel the phase delay of the estimation. The improvement can be observed from the euclidean norm of disturbance estimation error shown in Fig.10 too.

Figure 11 shows the frequency characteristic of $d \to \hat{d}$. It can be seen that the non-causal state observer is able to achieve a more ideal characteristic than the conventional method.

5. Conclusion

In this paper, a non-causal approach for state estimation is presented. This approach aims to improve state estimation by utilizing unused future data. It has been shown that compared to ordinary state observers only using past data, the proposed non-causal observer has improved estimation phase delay with the same estimation bandwidth. Non-causal state estimation is suitable for subjects such as ILC, where non-causal filtering is applicable. For future work, approaches for implementing it into ILC will be investigated.

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