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Model Order Selection in Robust-Control-Relevant System Identification

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Abstract: Robust control allows for guaranteed performance for a range of candidate models. The aim of this paper is to investigate the role of model complexity in the identification of model sets for robust control. A key point is that model quality and model complexity should be evaluated with respect to the control goal. Regularization using a worst-case control criterion in conjunction with a specific model uncertainty structure allows robust control of multivariable systems using accurate models with low complexity. Simulations confirm that the model order should be selected in view of the control objectives. Overall, the framework allows for systematic identification of model sets for robust control.

Keywords: Identification for control, Robust control, Motion control, Mechatronic systems, Frequency domain identification, Identification and control methods, Order selection

1. INTRODUCTION

The quality of any model should be assessed in view of its goal. For control, it is well known that models should be accurate for frequencies in the vicinity of the control bandwidth (Åström and Murray, 2010). For instance, evaluating the quality of models for next-generation motion control becomes particularly relevant due to increasingly complex systems (Oomen, 2018). This is a consequence of stringent demands regarding throughput and accuracy which leads to the situation where flexible dynamic behavior limits performance (Balas and Doyle, 1994). Consequently, flexible dynamic behavior needs to be actively controlled to achieve the desired performance, e.g. overactuation and oversensing (Van Herpen et al., 2014). The key difficulty in evaluating the quality of models for next-generation motion systems is to determine to what extend the flexible dynamic behavior needs to be modeled to achieve the desired performance.

In Schrama (1992) and Gevers (1993), the control goal is incorporated in the identification criterion of a nominal model aiming at a control-relevant nominal model. Typically, these methods are iterative and alternate between closed-loop identification and subsequent controller synthesis. However, a model is an approximation of reality, as a result, model errors are inevitable. Consequently, these methods do not have stability nor performance guarantees in case the designed controller is applied to the true system (Hjalmarsson et al., 1995).

Robust control takes modeling errors explicitly into account by designing a controller with guaranteed stability and performance based on a model set (Zhou et al., 1996).

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The requirements regarding the model set include (R1) encompass the true system, (R2) enable high-performance control, and (R3) limited complexity, e.g. in terms of the order of the generalized plant. Herein, (R3) is important for successful implementation as the order of the synthesized controller is often directly related to the order of the generalized plant (Zhou et al., 1996) as well as for numerical considerations (Datta, 2004).

The availability of reliable and systematic robust control algorithms has spurred the development of identification approaches of model sets for robust control. A first important step is taken by connecting the robust-control-relevant identification procedure to the robust control synthesis step (De Callafon and Van Den Hof, 2001). However, structured uncertainty is used which leads to a conservative plant set. As a consequence, it creates conservatism in the robust controller synthesis step (Doyle, 1982) thereby violating (R2). A key step is taken in view of (R2) in Oomen and Bosgra (2012) and Oomen et al. (2013) which allows the use of unstructured uncertainty in a non-conservative manner. This is achieved by connecting the robust-control-relevant identification procedure, the estimation of the size of the model uncertainty, and the robust controller synthesis step. However, the aspect of model order has not been addressed, hence (R3) it is not necessarily satisfied. In fact, iterative procedures may lead to a monotonic increase of model order.

Although important progress has been made in robust-control-relevant identification, at present model complexity is unaddressed. Essentially robust-control-relevant system identification enables modeling those phenomena that are important for control. However, it is unknown to what extent these phenomena need to be modeled to achieve the desired performance. For this reason, investigating the selection of an appropriate model order in view of the con-

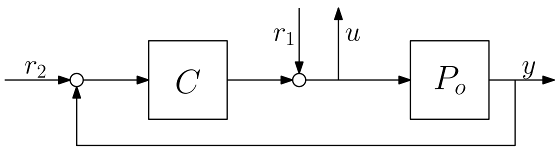


Fig. 1. Four-block feedback interconnection.

trol goal is crucial. This paper aims to develop a systematic approach for order selection in the identification of model sets, this pertains to both identification of nominal models, as well as weighting filters to shape the uncertainty. An early result regarding model complexity in robust-control-relevant identification is presented in Van Herpen et al. (2011). However, the distribution of model complexity over the nominal model and the weighting filters that shape the uncertainty results in an intractable trade-off. Also, an approach to address model complexity is lacking. Order selection is widely studied for the prediction error framework, see Stoica and Selen (2004) for an overview. Typically, these order selection rules use a probabilistic regularization term to select an appropriate model order. In this paper, a similar concept of a regularization term is adopted, yet fundamentally different through the use of a deterministic regularization term.

The main contributions of this paper are:

(C1) the development of a new method to determine the model order in view of the control goal.
(C2) development of a bound that transparently connects model uncertainty and model order.
(C3) the application of the proposed method on a relevant simulation example.

The outline of this paper is as follows. In Section 2, the general robust-control-relevant identification procedure is presented and the key problem addressed in this paper is discussed. In Section 3.1, the model structure used to develop the order selection procedure is presented. Then, in Section 3, the main result of this paper is presented, the model order selection procedure. The proposed algorithm is applied in Section 4 a simulation example that resembles the behavior of many mechatronic systems. The conclusion is presented in Section 5.

2. PROBLEM FORMULATION

2.1 Robust Control

A systematic approach to specify the control goal is to consider an $\mathcal{H}_\infty$-norm-based performance measure

$$\mathcal{J}(P,C) = \|WT(P,C)V\|_\infty \tag{1}$$

where W and V denote the weighting filters. The closed-loop feedback interconnection $T(P,C)$, see Fig. 1, is defined as

$$\begin{bmatrix} r_2 \\ r_1 \end{bmatrix} \mapsto \begin{bmatrix} y \\ u \end{bmatrix}, \begin{bmatrix} P \\ I \end{bmatrix} (I+CP)^{-1} [C \ I].$$

The control goal is to compute

$$C^{opt} = \arg\min_C \mathcal{J}(P_o, C)$$

where P_o denotes the true system. Since the true system is not known explicitly, a model set $\mathcal{P}$ is constructed which encompasses the true system such that $P_o \in \mathcal{P}$. For a given model set and controller, the worst-case performance measure is defined as

$$\mathcal{J}_{WC}(P,C) = \sup_{P\in\mathcal{P}} \mathcal{J}(P,C).$$

Since $P_o \in \mathcal{P}$, the following holds

$$\mathcal{J}(P_o,C) \le \mathcal{J}_{WC}(P,C). \tag{2}$$

Hence, the overbound (2) may be minimized to optimize the performance bound of the true system P_o. This leads to

$$C^{RP} = \arg\min_C \mathcal{J}_{WC}(\mathcal{P},C). \tag{3}$$

2.2 Robust-Control-Relevant Identification

The model set with respect to the robust controller is defined by a dual expression

$$\mathcal{P}^{RP} = \arg\min_{\mathcal{P}} \mathcal{J}_{WC}(\mathcal{P},C^{RP}). \tag{4}$$

However, the identification problem in (4) is intractable, since it depends on C^{RP} which is unavailable during the identification step. For this reason, the robust controller is approximated by the experimental controller C^{exp} which is known to stabilize the system and achieve a certain level of performance. This leads to the robust-control-relevant model set

$$\mathcal{P}^{RCR} = \arg\min_{\mathcal{P}} \mathcal{J}_{WC}(\mathcal{P},C^{exp}). \tag{5}$$

As is commonly encountered in identification for control, the initial controller affects the identification criterion. Consequently, if the distance between C^{exp} and C^{RP} is too large, then (3) and (5) may be solved iteratively. For many identification for control approaches, the performance does not necessarily converge. In contrast, for robust-control-relevant identification, model sets are employed. As a consequence, monotonous convergence is guaranteed (De Callafon and Van Den Hof, 1997; Garatti et al., 2010).

2.3 Problem Formulation

The robust-control-relevant model set is constructed around a nominal model $\hat{P}(n_x)$ of an order n_x and a norm-bounded model uncertainty Δ_u

$$\mathcal{P}(n_x) = \left\{ P \middle| P = \mathcal{F}_u(\hat{H}(\hat{P}(n_x)), \Delta_u), \Delta_u \in \boldsymbol{\Delta_u} \right\}.$$

Here, the upper linear fractional transformation (LFT) is defined as (Zhou et al., 1996)

$$\mathcal{F}_u(\hat{H}(\hat{P}(n_x)), \Delta_u) = \hat{H}_{22} + \hat{H}_{21}\Delta_u \left(I - \hat{H}_1 1\Delta_u\right)^{-1} \hat{H}_{12}$$

where $\hat{H}$ contains the nominal model $\hat{P}(n_x)$ and the uncertainty structure. The uncertainty set $\boldsymbol{\Delta_u}$ is defined as an $\mathcal{H}_\infty$-norm bounded perturbation

$$\boldsymbol{\Delta_u} = \{\Delta_u \in \mathcal{H}_\infty | \, \|\Delta_u\|_\infty \le \gamma\}.$$

The model order should be selected in view of the control goal. For instance, if a bandwidth is selected in the vicinity of the performance-limiting phenomena, e.g. resonances, the model should accurately describe the true system to achieve the desired performance. However, if a bandwidth is desired well ahead of these performance-limiting phenomena, the model may be less complex to enable the synthesis of a controller that achieves the desired performance. The key problem addressed in this paper is the development of an order selection procedure that breaks the trade-off between model order on the one hand and performance on the other hand while taking the control goal into account.

Order selection is widely studied for the prediction error framework (Stoica and Selen, 2004). The key idea is the introduction of a regularization-term which is often used in model order selection (Hjalmarsson et al., 1995)

$$\min_{n_x} \mathcal{J}_{WC}(\mathcal{P}^{RCR}(n_x), C^{exp}) + f(n_x). \tag{6}$$

Typically, the regularization term is a function that depends linearly on the model order (Stoica and Selen, 2004). In this research, the linear relation is adopted through a deterministic interpretation

$$\min_{n_x} \mathcal{J}_{WC}(\mathcal{P}^{RCR}(n_x), C^{exp}) + \phi n_x \tag{7}$$

where the parameter ϕ may be selected based on the goal. In the forthcoming, a connection is derived between the worst-case performance and the size of the uncertainty γ. Based on the novel connection, an order selection rule is proposed.

3. ORDER SELECTION FOR ROBUST-CONTROL-RELEVANT IDENTIFICATION

This section aims to develop an order selection that constitutes contribution (C1). To develop the order selection rule, a new relation is developed which transparently connects the worst-case performance associated with the model set with the size of the model uncertainty (C2). The particular choice of the model structure plays an important role.

3.1 Model Structure

Selecting an appropriate model structure is crucial for the identification of a model set that facilitates a transparent connection between the worst-case performance associated with the model set and the size of the model uncertainty. In the development of such a connection, the dual-Youla-Kučera structure is crucial.

Lemma 1. (Anderson, 1998) Let the nominal model $\hat{P}$ be internally stabilized by C^{exp}, and let the pairs $\{\hat{N}, \hat{D}\}$ and $\{N_c, D_c\}$ be Right Coprime Factorization (RCF) of $\hat{P}$ and C^{exp} respectively. Then, the dual-Youla-Kučera model set $\mathcal{P}^{dY}$ of all systems stabilized by C^{exp} is given by

$$\mathcal{P}^{dY} = \left\{ P \middle| P = \left(\hat{N} + D_c\Delta_u\right)\left(\hat{D} - N_c\Delta_u\right)^{-1} \right\}. \tag{8}$$

Note that infinitely many the coprime factorizations $\{\hat{N}, \hat{D}\}$ and $\{N_c, D_c\}$ exist of $\hat{P}$ and C^{exp}, e.g. the coprime factorizations are not unique. In the forthcoming, the non-uniqueness of the coprime factorizations is exploited to obtain a transparent connection between $J_{WC}\left(\mathcal{P}, C^{exp}\right)$ and the uncertainty bound γ.

Definition 2. (Oomen and Bosgra, 2012; Oomen et al., 2013) Let V be structured as $V = \mathrm{diag}(V_2, V_1)$ and let $\{\tilde{N}_e, \tilde{D}_e\}$ with $\tilde{N}_e = [\tilde{N}_{e,2}\tilde{N}_{e,1}]$ be a Left Coprime Factorization (LCF) with co-inner numerator of $[CV_2 \; V_1]$. The robust-control-relevant coprime factorization is defined as

$$\begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} \hat{P} \\ I \end{bmatrix} (\tilde{D}_e + \tilde{N}_{e,2}V_2^{-1}\hat{P})^{-1}.$$

Definition 3. (Oomen and Bosgra, 2012; Oomen et al., 2013) Let the weighting filter be structured as $W = \mathrm{diag}(W_y, W_u)$. The pair $\{N_c, D_c\}$ is an RCF with co-inner numerator of C^{exp} if it is an RCF of C^{exp} and if it satisfies

$$\begin{bmatrix} W_u N_c \\ W_y D_c \end{bmatrix}^* \begin{bmatrix} W_u N_c \\ W_y D_c \end{bmatrix} = I.$$

3.2 Order Selection Rule

By exploiting the non-uniqueness of the coprime factorizations in Definition 2 and 3, a transparent connection between the true performance and the uncertainty is derived.

Theorem 4. Let the model set be defined according to Lemma 1, with $\{\hat{N}, \hat{D}\}$ and $\{N_c, D_c\}$ defined by Definitions 2 and 3 respectively. Then, the worst-case performance is bounded by

$$\mathcal{J}_{WC}(\mathcal{P}^{RCR}, C) \leq \mathcal{J}(P_o, C) + 2\underbrace{\|\Delta_u\|_\infty}_{\gamma}.$$

Proof. The model set according to Proposition 1 with $\{\hat{N}, \hat{D}\}$ and $\{N_c, D_c\}$ defined by (2) and (3) respectively lead to the closed-loop LFT expression

$$\hat{M}^{RCR} = \begin{bmatrix} 0 & [\tilde{N}_{e,2} \; \tilde{N}_{e,1}] \\ \begin{bmatrix} W_y D_c \\ -W_u N_c \end{bmatrix} & WT(\hat{P}, C)V \end{bmatrix}.$$

Evaluating the worst-case performance associated with the robust-control-relevant model set and applying the triangular inequality leads to

$$\begin{aligned} \mathcal{J}_{WC}(\mathcal{P}^{RCR}, C^{exp}) &= \mathcal{F}_u\left(\hat{M}^{RCR}, \Delta_u\right), \\ &= \left\|\hat{M}_{22}^{RCR} + \hat{M}_{21}^{RCR}\Delta_u\hat{M}_{12}^{RCR}\right\|_\infty, \\ &\leq \left\|\hat{M}_{22}^{RCR}\right\|_\infty + \left\|\hat{M}_{21}^{RCR}\Delta_u\hat{M}_{12}^{RCR}\right\|_\infty. \end{aligned}$$

Note that the matrices $\hat{M}_{12}^{RCR}$ and $\hat{M}_{21}^{RCR}$ are co-inner. Hence, $\hat{M}_{12}^{RCR}$ and $\hat{M}_{21}^{RCR}$ are $\mathcal{H}_\infty$-norm preserving (Zhou et al., 1996). Next, observing that $\hat{M}_{22}^{RCR}$ equals $\mathcal{J}(\hat{P}, C)$ and $\|\Delta_u\|_\infty \leq \gamma$ leads to

$$\mathcal{J}_{WC}(\mathcal{P}^{RCR}, C) \leq \mathcal{J}(\hat{P}, C) + \gamma.$$

The final step is to apply the triangular inequality

$$\mathcal{J}(\hat{P}, C) \leq \mathcal{J}(P_0, C) + \gamma.$$

The size of the uncertainty is encountered twice, as the triangular inequality is applied twice.

The key advantage of the bound in Theorem 4 is that the bound solely depends on the performance $\mathcal{J}(P_0, C)$ and the size of the uncertainty. Hence, the order selection procedure can be based solely on the size of the uncertainty γ, since the performance $\mathcal{J}(P_0, C)$ is invariant under a change of model order. The relation derived in Theorem 4 combined with the order selection rule in (7) provides the following rule

$$\min_{n_x} \gamma + \phi n_x. \tag{9}$$

The essence of robust-control-relevant identification is to develop a transparent connection between the nominal model estimation and the size of the uncertainty. In fact,

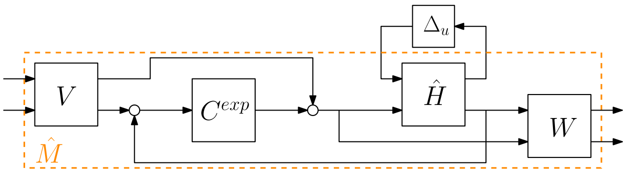


Fig. 2. Weighted closed-loop blockscheme.

the size of the uncertainty is used as an identification criterion of the nominal model (Oomen et al., 2013). Consequently, during the identification of the nominal model, the size of the uncertainty is minimized, hence γ is known. As a result, the key advantage of (9) with respect to (7) is that it only depends on the size of the model uncertainty which is computed during the estimation of the nominal model (Oomen and Bosgra, 2012; Oomen et al., 2013).

3.3 Discussion

Alternatively, the order cost in (7) may be computed based on the resulting robust controller C^{RP}, i.e.

$$\min_{n_x} \mathcal{J}_{WC}(\mathcal{P}^{RCR}(n_x), C^{RP}) + \phi n_x. \tag{10}$$

However, the optimization procedure in (10) requires for each order n_x the corresponding robust controller C^{RP} to be computed. This increases the computation time significantly. Instead, in the paper, an order selection rule is considered based on the experimental controller C^{exp}.

If the distance between C^{exp} and C^{RP} is too large, iterative identification and subsequent controller synthesis may be advantageous. In general, these iterative procedures do not necessarily converge. However, for the specific structure considered in this paper, monotonous convergence is guaranteed (Oomen and Bosgra, 2008).

The main concept in this paper is to select model order in view of the control goal. For instance, if a bandwidth is desired sufficiently far away from resonances, a less complex model may be sufficient to achieve the desired performance. However, if a bandwidth is desired in the vicinity of resonances, the model should accurately describe the system. The novel connection (4) between the worst-case performance associated with the robust-control-relevant model set enables to connect performance with the size of the uncertainty. As a result, the order selection rule proposed in this paper (9), takes the performance gain with respect to increasing model order into account. As a result, (9) selects the model order in view of the control goal.

4. EXAMPLE

This section shows the effectiveness of the robust-control-relevant identification procedure with the order selection procedure (9) by means of a simulation. In the simulation, a sixth-order mass-spring-damper system, Fig. 3, is considered which is a representative simulation example for many mechatronic/motion systems including the multivariable system in Fig. 6 (Oomen, 2018).

Table 1. Performance criteria for a bandwidth of 0.4 Hz for various model orders.

	γ	$\mathcal{J}(P_o, C^{exp})$	$\mathcal{J}_{WC}(\mathcal{P}^{RCR}, C^{exp})$
$n_x = 2$	$4.0 \cdot 10^{-2}$	6.476	6.497
$n_x = 4$	$6.5 \cdot 10^{-4}$	6.476	6.476
$n_x = 6$	$7.0 \cdot 10^{-10}$	6.476	6.476

4.1 Order Selection criterion

As a sixth-order motion system is considered, the system may be modeled by a second-order model (rigid-body mode only), fourth-order model, (rigid-body and one flexible mode) and, sixth-order model (rigid-body and two flexible modes). Hence, depending on the control objective, flexible modes may be added to the rigid-body model. If it is allowed to include flexible modes is determined by the order selection criterion (9). The parameter ϕ is selected as

$$\phi = \frac{2(\mathcal{J}(P_o, C^{exp}) + 2\gamma(n_{x,0}))}{2}$$

where $\gamma(n_{x,0})$ refers to the size of the uncertainty associated with the second-order model, $n_{x,0} = 2$. Hence, the tuning parameter in the order selection rule (9) is tuned by requiring at least 5% performance increase before it is allowed to increase the model order from a second-order model to a fourth-order model. Consequently, the tuning parameter depends on the control goal specified by the weighting filters, e.g. the model order is selected in view of the control goal. The weighting filters aim at a target bandwidth of $f_{bw} = 0.85$ Hz. The initial experimental controller achieves bandwidth of $f^*_{bw} = 0.85 \cdot f_{bw}$.

4.2 Results

The robust-control-relevant system identification procedure is performed for a second-, fourth- and, sixth-order model in Fig. 3. For the second-order model, only the rigid-body behavior is modeled. As a consequence, the model set is large except for frequencies in the vicinity of the bandwidth. This particular shape is the result of the robust-control-relevant coprime factorization. If a sixth-order model is selected, the entire behavior of the true system is incorporated in the nominal model. As a consequence, the model uncertainty is small.

The corresponding performance measures are depicted in Table 4.2. When investigating the performance measures, it is clear that the bound in Theorem 4 holds and it is sufficiently tight. When computing the cost for each model order, Fig. 4, it is clear that a second-order model achieves the minimum order cost. As a consequence, for this specific target bandwidth, adding model complexity does not result in a sufficient amount of performance gain.

The optimal model order is investigated for a large range of bandwidth scenarios in Fig. 5. If a bandwidth is selected sufficiently far away from the flexible dynamic behavior, a second-order model suffices for control purposes. On the other hand, if a bandwidth is selected slightly before the flexible dynamic behavior, a fourth-order model is optimal in view of (9). This is a consequence of the first flexible mode which is limiting the performance. For frequencies in the vicinity of the flexible dynamic behavior, a sixth-order model is required as the complete flexible dynamic behavior is limiting the performance. Overall, the simulation example confirms that the model order should be selected in view of the control goal.

4.3 Discussion

In this example, the weighting filters aim at a bandwidth f_{bw} and the initial experimental controller aims at a bandwidth of $0.85 \cdot f_{bw}$. Hence, the distance between the robust controller and the initial experimental controller is relatively small. However, for situations where an initial controller is available of which the distance with the desired robust controller is large, iterative identification and subsequent controller synthesis might be advantageous. For the model structure considered in this paper, monotonous convergence is guaranteed (Oomen and Bosgra, 2008).

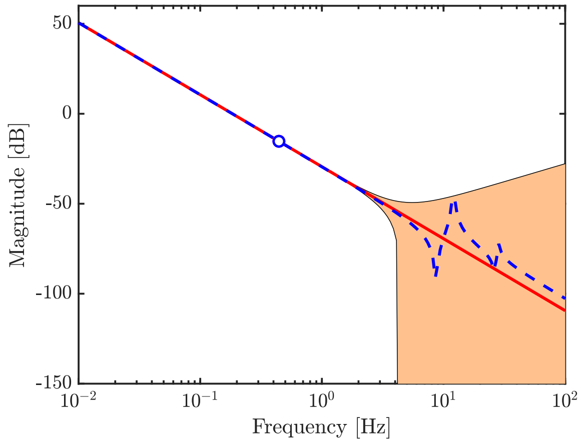
In the example, uncertainty is considered with respect to the nominal model. Hence, the uncertainty addresses the mismatch between the nominal model and the true system. However, uncertainty may also come from other sources. Often motion systems contain moving parts that affect the dynamics of the system (Wassink et al., 2005). As a consequence, the dynamics typically depends on the operating position. In Wassink et al. (2005), a position-dependent model is identified and a robust controller designed. However, it is unknown how to select the order of the model. Hence, it should be investigated how to select the model order in case of position-dependent behavior.

The main concept in this paper is to select the model order with respect to the control goal. In this example, an $\mathcal{H}_\infty$-norm based control objective is selected. It is emphasized that the optimal model order also depends on the selected control objective. Hence, changing the control objective leads to different results. For instance, to deal with unmeasured performance variables, a different control objective is required as shown in Oomen et al. (2009). However, it is unknown how to select the model order in case of unmeasured performance variables. It should be investigated how to select the model order in case of unmeasured performance variables.

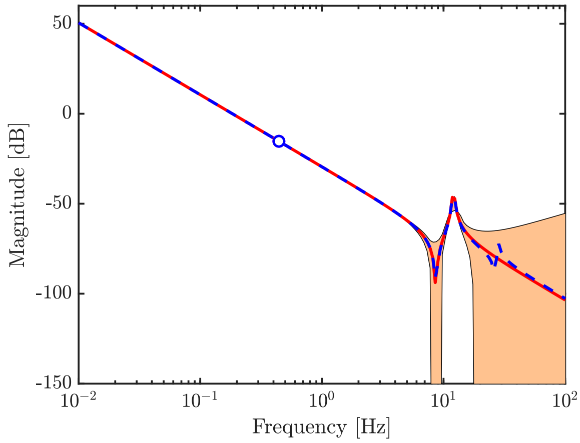
5. CONCLUSION

This paper provides a new perspective on model complexity in identification for robust control. Previous research in the identification for robust control focused on the selection of phenomena that are relevant for robust control. This paper extends to existing methods in the sense that it provides insight to what extend these phenomena need to be modeled to achieve the desired performance. This is achieved by selecting the model order in view of the control goal.

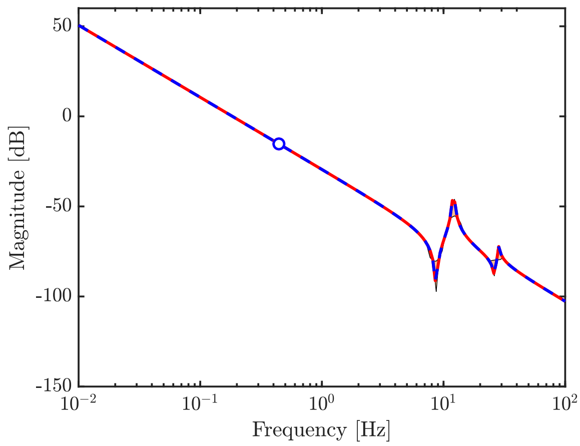
In the development of the order selection procedure, two important steps are taken. First, a transparent connection between the worst-case performance and the size of the model uncertainty is derived. Second, the connection is exploited through a regularization-based order selection criterion. The key mechanism of the proposed algorithm is to select the model order in view of the control goal. The effectiveness of the proposed algorithm is shown in the last section.



(a) $n_x^* = 2$



(b) $n_x^* = 4$



(c) $n_x^* = 6$

Fig. 3. Bode magnitude diagram of the nominal model $\hat{P}$ (—), the true system P_o (- - -) and the robust-control-relevant model set $\mathcal{P}^{RCR}$ (■). The desired bandwidth is indicated by (○).

Current research focuses on applying the robust-control-relevant identification framework in conjunction with the proposed order selection procedure to a multivariable motion stage depicted in Fig. 6.

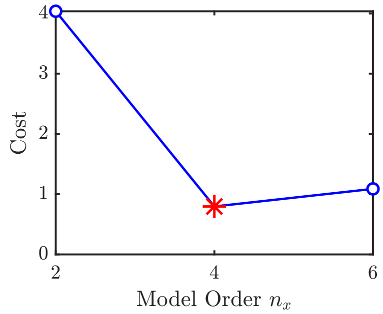


Fig. 4. Order selection cost as function of model order.

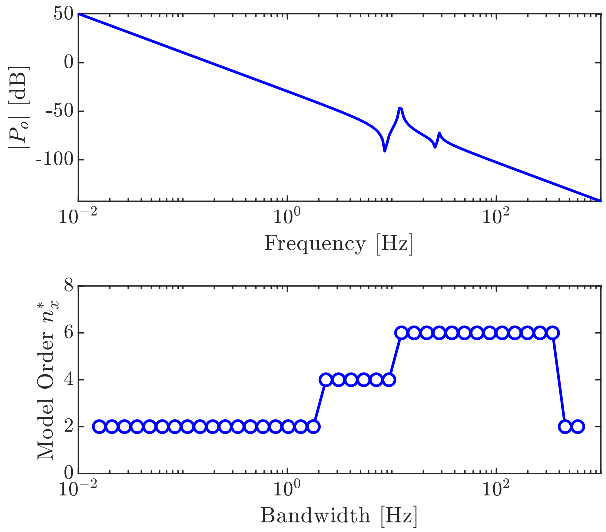


Fig. 5. Optimal model order as function of the desired bandwidth.

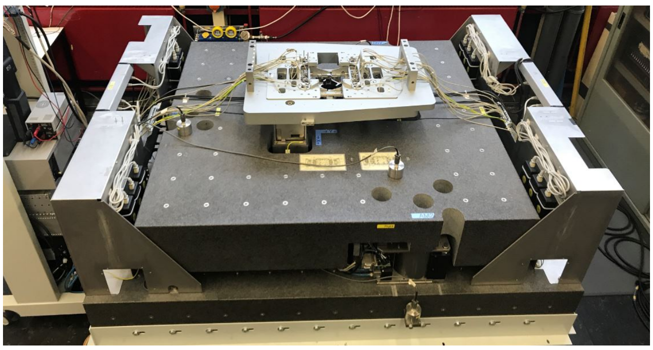


Fig. 6. Reticle stage setup at the motion lab of the TU/e.

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