Visualization of MIMO Uncertainty Structures for Robust Control

Paul Tacx[1] and Tom Oomen[1,2]
[1]Eindhoven University of Technology, Department of Mechanical Engineering
[2]Delft University of Technology, Delft Center for Systems and Control, Delft, The Netherlands
Email: p.j.m.m.tacx@tue.nl, t.a.e.oomen@tue.nl

1 Background

The availability of reliable and systematic robust control algorithms has spurred the development of uncertainty structures of multivariable model sets for robust control [1], [3]. Over de last decades uncertainty structures have developed from additive and multiplicative structures to more advanced structures specifically tailored for multivariable systems [2], [3]. The multivariable aspect and the more advanced uncertainty structures complicate uncertainty structure comparison, understanding, controller design, and performance analysis of uncertain systems.

2 Problem Formulation

The Bode plot is a critical tool for understanding, controller design, and performance analysis of control systems. In the scalar case, uncertainty structures for robust control reduce to a Möbius transformation from which the magnitude and phase can be computed. However, extending to multivariable systems is not straightforward, inducing the notion of phase of a multivariable uncertain system. The aim is to develop a unified approach for generating multivariable Bode plots for both the magnitude and phase of multivariable uncertain systems.

3 Approach

The key observation is that the Bode plot of scalar systems is based on the polar decomposition which naturally connects the Nyquist plot and Bode magnitude and phase plot. The key idea is to extend the polar description to the multivariable case which leads to a multivariable magnitude and phase definition [4]. The resulting multivariable magnitude and phase are extended to the uncertain case by reformulation through the S-procedure to a feasibility problem.

4 Results

The developed approach is applied to a multivariable motion system based on the uncertainty structure in [3]. The resulting multivariable Bode magnitude and phase are shown in Fig. 1, 2. The figures reveal that for frequencies in the vicinity of the bandwidth, the model set is tight. This specific behavior is attributed to the control-relevant structure. Overall, the example shows that the proposed approach offers additional insight and understanding by generating multivariable Bode plots of uncertain systems.

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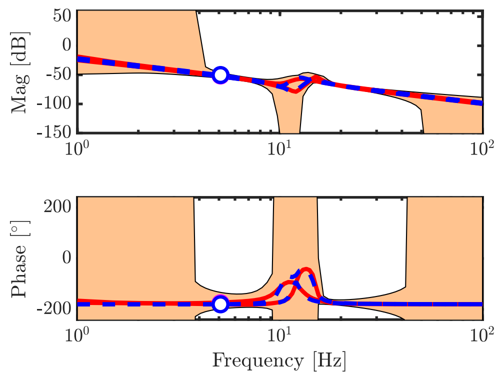

Figure 1: Multivariable Bode magnitude (top) and phase (bottom). True system P_o (—), nominal model $\hat{P}$ (—), model set ( ), and target bandwidth (○).

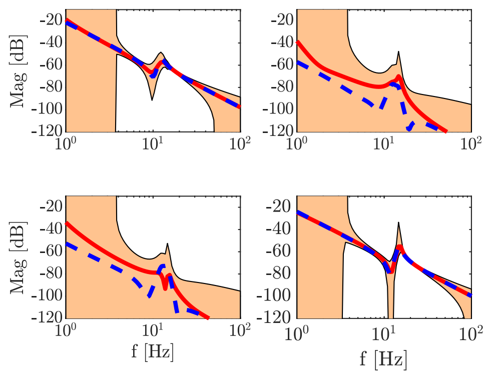

Figure 2: Element-wise Bode magnitude. True system P_o (—), nominal model $\hat{P}$ (—), model set ( ).

References

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