Accurate $\mathscr{H}_\infty$-Norm Estimation via Finite-Frequency Norms of MIMO Local Parametric Models

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Paul Tacx & Tom Oomen
p.j.m.m.tacx@tue.nl

Background

Identification for control:

Given model $\hat{P}$, uncertainty Δ

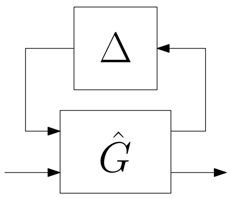
$\Longrightarrow$ Model set $\mathscr{P}$

Design controller that performs well for $\forall P \in \mathscr{P}$.

Problem

Model uncertainty bound γ is crucial for performance:

$\|\Delta\|_\infty \ll \gamma$: Conservative
$\|\Delta\|_\infty > \gamma$: No guarantees
$\|\Delta\|_\infty \leq \gamma$: Guarantees

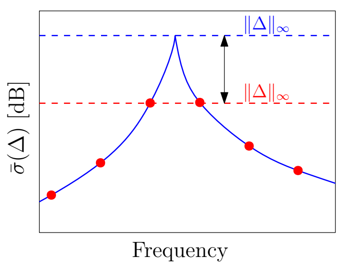


Aim: Accurate and reliable $\mathscr{H}_\infty$-norm estimation

Traditional Approach

FRF-based algorithms: at-grid estimation

$\Longrightarrow$ inter-grid errors



This Research

Local Modeling Techniques

Key idea: include inter-grid behaviour by local modeling techniques [1], [2]

$$Y(k) = \Delta(\xi_k)U(k) + T_\Delta(\xi_k) + V(k)$$

$\Delta(\xi_k)$ and $T_\Delta(\xi_k)$: smooth over frequency

Exploit local smoothness: approximate Δ, T_Δ locally by parametric $\tilde{\Delta}_k$, $\tilde{T}_{\Delta,k}$

$$\Delta(\xi_{k+r}) \approx \tilde{\Delta}(\xi_{k+r}) = D_k^{-1}(r)N_K(r) \quad \text{(LMFD)}$$

$$T_\Delta(\xi_{k+r}) \approx \tilde{T}_{\Delta,k}(\xi_{k+r}) = D_k^{-1}(r)M_K(r) \quad \text{(LMFD)}$$

valid on domain Ω_k

$\mathscr{H}_\infty$-norm Estimation

Key idea: Estimating global $\mathscr{H}_\infty$ norm through local $\mathscr{L}_\infty$-norms of local models:

Step 1 Finite-frequency $\mathscr{L}_\infty$ norm: $\gamma_k = \sup_{\omega\in\Omega_k} \tilde{\Delta}_k(\xi)$
Conversion to LMI using generalized KYP lemma [3]:

$$F(\tilde{\gamma}_k) = \begin{bmatrix} A & B \\ E & 0 \end{bmatrix}^* (\Phi \otimes P + \Psi \otimes Q) \begin{bmatrix} A & B \\ E & 0 \end{bmatrix} + \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & -\tilde{\gamma}_k^2 I \end{bmatrix} \begin{bmatrix} C & D \\ 0 & I \end{bmatrix} \prec 0.$$

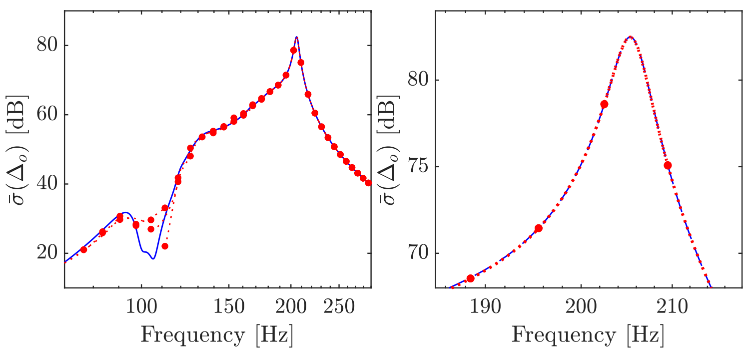
Accurate and reliable finite-frequency $\mathscr{L}_\infty$-norm computation

Step 2 Global $\mathscr{H}_\infty$ norm: $\|\Delta\|_\infty \approx \max_k \gamma_k$

Results: 2x2 MIMO System

2x2 MIMO Mechanical system
Required: $\|\Delta\|_\infty$
$\Longrightarrow$ only FRF data available

	$\|\Delta\|_\infty$ [dB]
True	82
Traditional	78
This poster	82



Accurate $\mathscr{H}_\infty$-norm estimate using limited data

References

[1] R. Pintelon, J. Schoukens, G. Vandersteen, and K. Barbé, Estimation of nonparametric noise and FRF models for multivariable systems – part I: Theory, Mechanical Systems and Signal Processing 24 (2010), no. 3, 573 – 595.

[2] E. Geerardyn, and T. Oomen, A local rational model approach for $\mathscr{H}_\infty$ norm estimation: With application to an active vibration isolation system, Control Engineering Practice, vol. 68, no 1, pp. 63-70, 2017.

[3] T. Iwasaki and S. Hara, Generalized KYP lemma: Unified frequency domain inequalities with design applications, IEEE Transactions on Automatic Control, vol. 50, no. 1, pp. 41–59, 2005.