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Hysteresis Feedforward Compensation: A Direct Tuning Approach using Hybrid-MEM-Elements

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Abstract— **Compensation of hysteresis enables substantial performance improvements, e.g., in case of piezoelectric actuators. The aim of this paper is to develop a systematic manual tuning approach for a feedforward controller that compensates for hysteresis phenomena. Modelling the hysteresis phenomena using a hybrid-memory-element enables the determination of a feedforward controller for which the influence of each of the feedforward parameters can be distinguished in the error during a closed-loop experiment. This allows for a direct systematic approach to tune the feedforward parameters resulting in a feedforward controller relevant for closed-loop control without the need for an extensive identification procedure.**

I. Introduction

Feedforward control can effectively reject known disturbances, e.g., a desired trajectory before these affect the system. In general, a feedforward controller consists out of the inverse model of the system [1], [2]. For example, in linear mechanical systems, Newton's law $F = ma$ can be used, the parameter m, representing the mass of the system, can easily be determined in a closed-loop setting using a direct manual tuning approach, see e.g., [3] for tuning guidelines, and [4] for an automated tuning algorithm. The advantage of these direct approaches is a feedforward controller which is relevant for closed-loop control by design.

Besides feedforward for linear systems, feedforward can also be effectively applied to compensate for nonlinear phenomena such as Coulomb friction [5]. Interestingly, the inverse model of Coulomb friction can be uniquely determined. By parameterising the feedforward linear in the parameters, it can be efficiently tuned in a user-friendly manner [3].

Nonlinear hysteresis phenomena can significantly deteriorate the tracking performance, e.g., in systems with piezoelectric actuators [6], [7], [8]. A wide variety of models have been developed to model the hysteresis phenomena, including the Prandtl-Ishlinskii model [9], [10], the Ramberg-Osgood model [11], [12], the Maxwell-Slip model [13], the Duhem model [14], and the Preisach model [15].

A key challenge to determine a feedforward controller to compensate for the hysteresis phenomena is the non-unique input-output behaviour. Due to the history dependence of the hysteresis effect inverting this non-unique input-output mapping is non-trivial. Despite this several feedforward controllers to compensate hysteresis exist, see, e.g., indirect approaches in [9], [16], [17] where first the hysteresis is explicitly modelled and compensated in two steps. To simplify the identification procedure a linear approximation of the hysteresis model can be identified which allows one to use existing linear system inversion techniques [18]. Due to the linear approximation modelling errors, and thereby an incorrect feedforward, are inevitable. This approach is further extended in [19] where a hybrid-memory(MEM)-element [20] enables a unique inverse of the Ramberg-Osgood hysteresis model [19], yet the identification of the hybrid-MEM-element is of comparable difficulty.

Although many approaches are available to model and compensate hysteresis phenomena, at present first an intensive identification procedure of a hysteresis model is required to determine its feedforward controller. The aim of this paper is to develop a hybrid-MEM-element to uniquely invert the Prandtl-Ishlinskii model and exploiting this as a feedforward controller to compensate for hysteresis phenomena. In sharp contrast to [19], the Prandtl-Ishlinskii model facilitates a systematic manual tuning approach which directly determines the parameters of the feedforward controller. This approach eliminates the need for an intensive identification of the hysteresis model, and the resulting feedforward controller is by design relevant in closed-loop control.

This leads to the following contributions of this paper.

C1 A hybrid-MEM-element is developed to model Prandtl-Ishlinskii type of hysteresis. (Section III)

C2 A feedforward controller to uniquely invert the hysteresis phenomena is determined which exploits the mapping of the hybrid-MEM-element. (Section IV)

C3 The systematic direct tuning approach to determine the required mapping of the MEM-element necessary for the feedforward controller. (Section II and V)

II. Problem Formulation

A. Feedforward for Servo Control

In servo control the goal is to perform a prespecified task, i.e., let an output y of a system G follow a desired trajectory y_d. A typical control architecture to achieve this is shown in Fig. 1, see, e.g, [3], where K and F are the feedback and feedforward controllers. When all systems are linear time-invariant (LTI),

$$e = S\underbrace{(1-GF)}_{\text{feedforward}}y_d - \underbrace{S}_{\text{feedback}}v - \underbrace{S}_{\text{feedback}}\eta, \tag{1}$$

where $S = (1+GK)^{-1}$, v a disturbance affecting the system and η a measurement noise. The goal of feedforward is to obtain an input signal u_{ff} for the plant G such that

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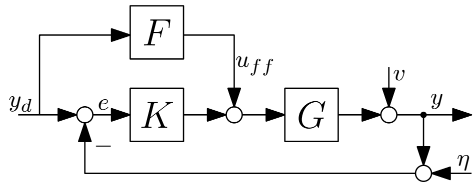

Fig. 1. Typical control architecture for servo control.

the desired trajectory y_d is followed. This is achieved if the feedforward controller F is an exact inverse of the system G. The goal of the feedback controller is attenuating the disturbance v and the residual of $(1 - GF)y_d$ due to incomplete knowledge of G in the design of F.

In many engineered systems, the feedforward controller F can be determined by manually tuning its parameters during closed-loop experiments. This results in a feedforward controller which eliminates the need for an identification procedure of the system G and is relevant for closed-loop control by design. Next, a feedforward tuning approach for a system consisting out of a linear spring is exemplified. This insight is used in the remainder of this paper where a new procedure to manually tune the feedforward controller of a system with hysteresis is developed.

The following procedure can be applied to systems with many feedforward parameters, for illustration only a single parameter is considered. Consider $G(s) = k$ with $k \in \mathbb{R}_{>0}$, representing a mechanical spring. Assume the controller $K(s) = \frac{\beta}{s+\alpha}$, i.e., a first-order lowpass filter. From (1),

$$e(s) = \frac{s+\alpha}{s+\alpha+\beta k}(1 - kF(s))y_d(s). \tag{2}$$

A feedforward controller given by

$$F(s) = c \tag{3}$$

with $c = \frac{1}{k}$ ensures $(1 - GF) = (1 - kc) = 0$.

A desired trajectory y_d for which the asymptotic error, i.e., $\lim_{t\to\infty} e(t)$, scales linearly with the value of $(1 - kc)$ allows for a systematic manual tuning approach, to find the parameter c in (3), based on this time-domain error signal. Consider the following result for trajectory design.

Theorem 1 (Final value theorem [21]) *Given a function $f(t)$ and its Laplace transform $F(s)$. Suppose that every pole of $F(s)$ is either in the open left half plane or at most a single pole at the origin. Then,*

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s). \tag{4}$$

Then it follows that

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot e(s) = s\frac{\alpha}{\alpha+\beta k}(1 - kc)y_d(s) \tag{5}$$

Choosing $y_d(s) = 1/s$, i.e., a step function in the time domain, leads to a final value of the error (5) which scales linearly with difference $(1 - kc)$. This leads to the following systematic manual tuning procedure.

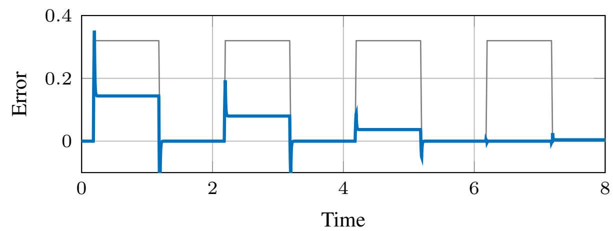

Fig. 2. Error signal, in blue (—), obtained from a closed-loop experiment as described in Procedure 1 with a a scaled version of the desired trajectory depicted in gray (—). After each constant segment the value of c is increased leading to a $e = 0$, i.e., the correct value of c is found.

Procedure 1 (Feedforward Tuning for Linear Spring)

1) Choose $y_d(t)$ piecewise linear, e.g., as Fig. 2. Apply $y_d(t)$ with a controller $K(s) = \frac{\beta}{s+\alpha}$. Choose the length of each step sufficiently long to eliminate transients.
2) Start with the feedforward parameter $c = 0$.
3) Gradually increase the parameter c, and notice the decrease of the asymptotic value of the error at the end of each constant position part, i.e., the value given by (5). Stop increasing the value c at the moment the error increases again, see Fig. 2 where the error data of a few tuning steps are depicted.
4) The optimal feedforward is given by the value of c for which the error is minimized.

The feedforward controller can be extended to compensate for other components [3], e.g., a mass, friction, Coulomb friction. The reference trajectory to tune the parameters corresponding to these components should be altered accordingly, e.g., when considering a mass system, i.e., $G(s) = \frac{1}{ms^2}$, and $F(s) = cs^2$, the asymptotic error (5) is given by

$$\begin{aligned}\lim_{t\to\infty} e(t) &= s \lim_{s\to 0} \frac{ms^3 + m\alpha s^2}{ms^3 + m\alpha s^2 + \beta}\left(1 - \frac{c}{m}\right) y_d(s) \\ &= s^3 \frac{m\alpha}{\beta}\left(1 - \frac{c}{m}\right) y_d(s)\end{aligned} \tag{6}$$

from which it follows that by choosing the trajectory $y_d(s) = s^3$, i.e., constant acceleration, the asymptotic error scales linearly with the difference $(1 - \frac{c}{m})$.

B. Hysteresis

Hysteresis can have significant impact on control performance, e.g., when using a piezoelectric actuator, [18]. A wide variety of models and corresponding compensation methods are available to represent hysteresis, see Table I for a brief overview. As is shown in this paper the Prandtl-Ishlinskii model [9], [16] has a favourable structure which allows for a control-relevant identification procedure.

The Prandtl-Ishlinskii model is a linear weighted superposition of a finite number of play operators, i.e.,

$$y(t) = \sum_{j=1}^{N} w_j H_{r_j}(u(t)), \tag{7}$$

with N the number of play operators and w_j $j \in \{1, ..., N\}$ the weighting corresponding to the play operator H_{r_j} $j \in \{1, ..., N\}$. For input functions $u(t)$, that are monotone, in each interval $[t_i, t_{i+1})$ of a partition $0 = t_0 \leq t_1 ... \leq t_i \leq$

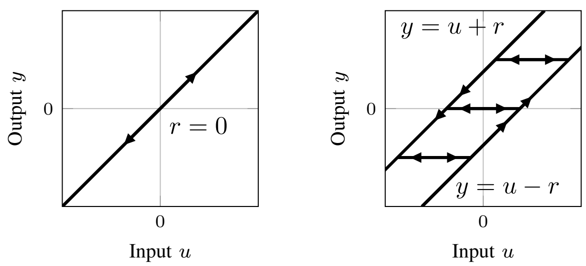

Fig. 3. Depiction of a play operator with $r = 0$ (left) and $r \in \mathbb{R}_{>0}$ (right)

TABLE I

BRIEF OVERVIEW OF METHODS TO COMPENSATE FOR HYSTERESIS

Model	Reference	Identification
Prandtl-Ishlinskii	[16], [9]	independent of controller
Preisach	[17]	independent of controller
Ramberg-Osgood	[19]	independent of controller
Prandtl-Ishlinskii	this paper	control relevant

$t \leq t_{i+1} ... \leq t_M$ and for a given threshold $r \geq 0$, the output of the play operator H_r is defined by

$$y(t) = H_r(u(t)) = \max(u(t) - r, \min(u(t) + r, y(t_{i-1})), \quad (8)$$

for each section $t \in [t_{i-1}, t_i)$, and initial condition $y(t_0) = \max(u(0) - r, \min(u(0) + r, 0)))$.

Example 2 Consider a Prandtl-Ishlinskii model with four play operators, i.e., $N = 4$, and threshold values $r_1 = 0$, $r_2 = 1$, $r_3 = 2$ and $r_4 = 3$, with corresponding weightings $w_1 = 1$, $w_2 = 1.5$, $w_3 = 2$, and $w_4 = 2.5$. When applying a triangular input signal u as in Fig. 4 the output y is given in Fig. 5. This leads to the hysteresis loop depicted in Fig. 6.

C. Problem Formulation

Given sytems that can be described by the Prandtl-Ishlinskii model in (7), develop a direct feedforward tuning approach, involving the parametrization of the inverse model as feedforward controller, the experiment design for a systematic manual tuning experiment, and a parameter adaptation strategy similar to Procedure 1.

III. HYSTERESIS AS A MEMORY ELEMENT

A. Description of memory-element

A general memory (MEM)-element

$$y_m(t) = M(p_m(t))u_m(t) \quad (9)$$

is considered, with input $u_m(t)$ and output $y_m(t)$. Here $M : \mathbb{R} \to \mathbb{R}$ is a function of

$$p_m(t) = p_m(t_i) + \int_{t_i}^{t} g(u_m(\tau))\,\mathrm{d}\tau, \forall t \in [t_i, t_{i+1}), \quad (10)$$

where $g : \mathbb{R} \to \mathbb{R}$. This general description of a hybrid MEM-element captures all the standard MEM-elements as introduced in [22]. The signal $p_m(t)$ represents the memory of the input, which can be interpreted as momentum. The momentum $p_m(t)$ in a hybrid system MEM-element can be reset at a time instant t_i to a value depending on $p_m(t_i)$ and $u_m(t_i)$ [20], i.e.,

$$p_m(t_i) = f(p_m(t_i^-), u_m(t_i^-)), \quad (11)$$

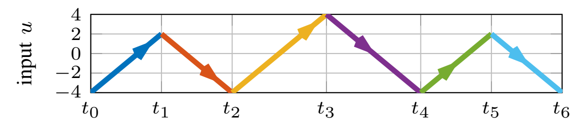

Fig. 4. Piecewise monotonous input $u(t)$ with monotonicity partitions $[t_0, t_1)$, $[t_1, t_2)$, ..., $[t_5, t_6)$ indicated with different colors.

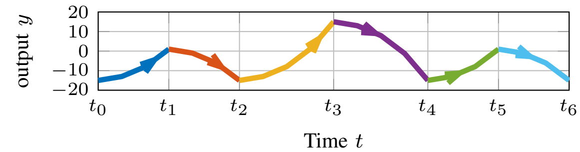

Fig. 5. Output $y(t)$ of a Prandtl-Ishlinskii model with parameters as given in Example 2 with input signal as depicted in Fig. 4. Each color corresponds to a monotonic partition of the input signal as in Fig. 4

with $p_m(t_i^-) = \lim_{s \uparrow t_i} p_m(s)$ and $u_m(t_i^-) = \lim_{s \uparrow t_i} u_m(s)$. The time instances of the resets, $t_i, i \in \mathbb{N}$, can depend on the state and input of the system. This property will be exploited to reset the memory at the start of each monotonic interval of the input to model the Prandtl-Ishlinskii model as a hybrid-MEM-element.

B. Prandtl-Ishlinskii model as a hybrid-MEM-element

In this section, the Prandtl-Ishlinskii model (7) is rewritten as a hybrid-MEM-element. First, consider the input-output behaviour from the time derivative of the input to the time derivative of the output for a single play operator (8), i.e.,

$$\dot{y}(t) = \begin{cases} 0 \text{ if } \; -r < u(t) - y(t_i) < r, \\ \dot{u}(t) \text{ otherwise}, \end{cases} \quad (12)$$

for $t \in [t_i, t_{i+1})$. The following auxiliary result is obtained.

Lemma 3 *Consider the play operator* (8)*. The input-output behaviour from the time derivative of the input, i.e.,* $\dot{u}$*, to the time derivative of the output, i.e.,* $\dot{y}$*, is equivalent to the hybrid-MEM-element* (9) *with* $u_m(t) = \dot{u}(t)$*,* $y_m(t) = \dot{y}(t)$*,*

$$M(p_m(t)) = \begin{cases} 0 \text{ if } \; -r < p_m(t) < r, \\ 1 \text{ otherwise }, \end{cases} \quad (13)$$

where the momentum $p_m(t) = u(t) - y(t_i)$*,* $t \in [t_i, t_{i+1})$*, is determined from* (10) *with* $g(u_m) = u_m$*, and momentum reset* (11) *with*

$$f(p(t_i^-), u_m(t_i^-)) = \begin{cases} -r & \text{if} \quad p(t_i^-) \leq -r, \\ p(t_i^-) & \text{if} \; -r < p(t_i^-) < r, \\ r & \text{if} \quad r \leq p(t_i^-). \end{cases} \quad (14)$$

Proof Consider a play operator (8) with threshold $r \in \mathbb{R}_{\geq 0}$. First, rewrite (8), to clearly distinguish between the conditions for each of the three possible phases in the interval $t \in [t_i, t_{i+1})$, i.e., $y(t) = u(t) + r$, $y(t) = u(t) - r$ and $y(t) = y(t_i)$,

$$y(t) = \begin{cases} u(t) + r \text{ if } u(t) + r \leq y(t_i), \\ u(t) - r \text{ if } u(t) - r \geq y(t_i), \\ y(t_i) \text{ otherwise }, \end{cases} \quad (15)$$

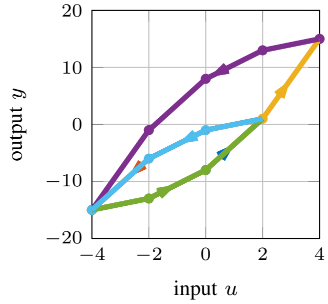

Fig. 6. Hysteresis loop of a Prandtl-Ishlinskii model with parameters as given in Example 2 with input signal as depicted in Fig. 4. Each color corresponds to a monotonic partition of the input signal as in Fig. 4. The dots (•) indicate the time instances where the input exceeds the threshold value of one of the play operators.

in each interval $t \in [t_i, t_{i+1})$. With its differential form

$$\dot{y} = \begin{cases} 0 \text{ if } u(t) - r < y(t_i) < u(t) + r, \\ \dot{u} \text{ otherwise,} \end{cases} \tag{16}$$

in each interval $t \in [t_i, t_{i+1})$. By defining the momentum

$$p(t) = u(t) - y(t_i) \tag{17}$$

on each interval $t \in [t_i, t_{i+1})$ allows to rewrite (16) as $\dot{y}(t) = M(p(t))\dot{u}(t)$ with M given by (13). The momentum $p(t)$ can be computed using $p(t) = p(t_i) + \int_{t_i}^{t} \dot{u}(\tau)\,\mathrm{d}\tau$ in each interval $t \in [t_i, t_{i+1})$ from which it follows that $g(u) = u$ in (10). Moreover from (17) it follows that, $p(t_i) = u(t_i) - y(t_i) = u(t_i^-) - y(t_i^-)$ with $u(t_i^-) = \lim_{s \uparrow t_i} u(s)$ and $y(s) = \lim_{s \uparrow t_i} y(s)$. Substituting $y(t)$ from (15) leads to

$$\begin{aligned} p(t_i) &= u(t_i^-) - y(t_i^-) \\ &= \begin{cases} u(t_i^-) - u(t_i^-) - r \text{ if } u(t_i^-) + r \leq y(t_i^-), \\ u(t_i^-) - u(t_i^-) + r \text{ if } u(t_i^-) - r \geq y(t_i^-), \\ u(t_i^-) - y(t_i^-) \text{ otherwise,} \end{cases} \end{aligned} \tag{18}$$

which can be rewritten as $p(t_i) = \begin{cases} -r \text{ if } p(t_i^-) \leq -r, \\ r \text{ if } p(t_i^-) \geq r, \\ p(t_i^-) \text{ otherwise,} \end{cases}$ which shows that the mapping f in (11) is given by (14). This completes the proof. ■

The next step in the approach is to recast the Prandtl-Ishlinskii model as a single hybrid-MEM-model, by employing the fact that it is a weighted sum of play operators. To accommodate for N play operators, the momentum signal consists out of N elements, i.e., $p_m(t) \in \mathbb{R}^N$ for all $t \in \mathbb{R}_{\geq 0}$, where each element represents the momentum signal of one of the play operators. This leads to the following result.

Theorem 4 *Given Prandtl-Ishlinskii model* (7) *with weights $w_j, j \in \{1, .., N\}$ and thresholds $r_j, j \in \{1, ..., N\}$, with input $u(t)$ and output $y(t)$. Then the input-output behaviour from the time derivative of the input $\dot{u}(t)$ to the time derivative of the output $\dot{y}(t)$ is equivalent to the hybrid-MEM-element* (9) *with $u_m(t) = \dot{u}(t)$, $y_m(t) = \dot{y}(t)$, and $p_m(t) \in \mathbb{R}^N$*

$$M(p_m) = \sum_{j=1}^{N} w_j m_j(p_m), \text{ where,} \tag{19}$$

$$m_j(p_m) = \begin{cases} 0 \text{ if } -r_j < p_{m_j} < r_j, \\ 1 \text{ otherwise,} \end{cases} \tag{20}$$

with $p_{m_j}(t)$ the j^{th} element of the momentum vector $p_m(t)$. The momentum vector $p_m(t)$ is determined from (10) *with $g(u_m) = \begin{bmatrix} u_m & \dots & u_m \end{bmatrix}^T$, and $f(p) = \begin{bmatrix} f_1(p_1(t)) & \dots & f_N(p_N(t)) \end{bmatrix}^T$, with*

$$f_j(p_j(t_i^-)) = \begin{cases} -r_j & \text{if} \quad p_j(t_i^-) \leq -r \\ p_j(t_i^-) & \text{if } -r_j < p_j(t_i^-) < r_j \\ r_j & \text{if} \quad r_j \leq p_j(t_i^-) \end{cases} \tag{21}$$

Proof The proof has two parts. Part 1 shows that the Prandtl-Ishlinskii model can be written as the sum of hybrid-MEM-elements. Part 2 shows that this sum of hybrid-MEM-elements can be written as a single hybrid-MEM-element.

Part 1: The differential form of the Prandtl-Ishlinskii model (7) is given by $\dot{y}(t) = \sum_{j=1}^{N} w_j \dot{y}_j(t)$ with $\dot{y}_j(t)$ the time-derivative of the output of the j^{th} play-operator. From Lemma 3 it follows that the time-derivative of the output of a play operator is given by $\dot{y}_j(t) = m_j(p_j(t))\dot{u}(t)$ where m_j and $p_j(t)$ are the mapping M and momentum $p(t)$ corresponding the j^{th} play-operator as defined in Lemma 3. This shows that the Prandtl-Ishlinskii model can be written as a sum of hybrid-MEM-elements.

Part 2: To show that this sum can be captured by a single mapping M define the momentum vector $p_m(t) \in \mathbb{R}^N$, as $p_m(t) = \begin{bmatrix} u(t) - y_0(t_{i-1}) & \dots & u(t) - y_N(t_{i-1}) \end{bmatrix}^T$ which can be determined using (10) with $g(\dot{u}) = \begin{bmatrix} \dot{u} \dots \dot{u} \end{bmatrix}^T$, and each element of the momentum vector is reset at each t_i following (14), i.e., the reset given by (21). This allows one to write the Prandtl-Ishlinskii model as (9) where M is given by (19). This completes the proof. ■

Given that the Prandtl-Ishlinskii model can be written as a hybrid-MEM-element allows one to define a feedforward controller as a unique inverse of this hybrid-MEM-element, see Section IV.

IV. Feedforward for Prandtl-Ishlinskii Hysteresis via MEM modelling

Exploiting the result of Theorem 4 which shows the Prandtl-Ishlinskii model can be written as a hybrid-MEM-element, the feedforward controller can be defined as the unique inverse of the hybrid-MEM-element, leading to Contribution C2 of this paper.

Assumption 5 *The play operator with the lowest threshold value, i.e., r_1, in a Prandtl-Ishlinskii model satisfies $r_1 = 0$.*

This assumption allows one to develop a feedforward controller leading to $y(t) = y_d(t)$ for any reference $y_d(t)$. At least one threshold value $r = 0$, ensures that there does not exist an input $u(t)$ such that all play operators are in their deadzone, i.e., a change in the input always will lead to a change in the output.

Given this assumption the following result is obtained exploiting Theorem 6 from [19].

Theorem 6 *Consider a hysteretic behaviour given by* (7) *with input* $u(t)$*, output* $y(t)$*, consisting of a weighted sum of* $N \in \mathbb{N}$ *play operators with threshold values* r_i *and weightings* w_i*,* $i \in \{1, ..., N\}$ *satisfying Assumption 5. Moreover, consider the feedforward controller given by*

$$\dot{u}_{ff}(t) = \frac{1}{\sum_{j=1}^{N} w_j m_j(p_{ff}(t))} \dot{y}_d(t), \tag{22}$$

with $\dot{y}_d(t)$ *the time derivative of the desired trajectory,* $\dot{u}_{ff}(t)$ *the generated input rate signal, and*

$$m_j(p_{ff}(t)) = \begin{cases} 0 \text{ if } -r_j < p_{ff_j}(t) < r_j, \\ 1 \text{ otherwise,} \end{cases} \tag{23}$$

with $p_{ff_j}(t)$ *the* j^{th} *element of the momentum* $p_{ff}(t) \in \mathbb{R}^N$*, which can be determined from*

$$p_{ff}(t) = p_{ff}(t_i) + \int_{t_i}^{t} \begin{bmatrix} \dot{u}_{ff}(\tau) & ... & \dot{u}_{ff}(\tau) \end{bmatrix}^T \mathrm{d}\tau \tag{24}$$

and $p_{ff}(t_i) = f(p_{ff}(t_i^-))$*, where* $f(p) = \begin{bmatrix} f_1 & ... & f_N \end{bmatrix}^T$ *with*

$$f_j(p_{ff_j}(t_i^-)) = \begin{cases} -r_j & \textit{if} \quad p_{ff}(t_i^-) \leq -r \\ p_{ff_j}(t_i^-) & \textit{if} \quad -r_j < p_{ff}(t_i^-) < r_j \\ r_j & \textit{if} \quad r_j \leq p_{ff}(t_i^-) \end{cases} \tag{25}$$

Applying this input rate to the the piezoelectric actuator with $y(0) = y_d(0)$*, and* $u(0) = 0$*, leads to the output trajectory* $y(t) = y_d(t)$ *for all* $t \in \mathbb{R}_{\geq 0}$*.*

From this result, it is concluded that when both the threshold values r_j and weights w_j for all $j \in \{1, ..., N\}$ are known the hysteresis can be fully compensated. In the next section, a tuning approach is developed that can determine these unknown parameters.

V. Hysteresis Feedforward Tuning

In this section, a systematic manual tuning approach is introduced to find the parameters necessary to implement the feedforward controller as introduced in Theorem 6. This approach is an extension Procedure 1.

To determine the systematic tuning approach that directly leads to a control relevant feedforward controller consider closed-loop experiments of the control setup in Fig. 1 where the system G has hysteretic behaviour, the controller $K(s) = \frac{\beta}{s+\alpha}$ stabilizes the system and a desired trajectory y_d is to be designed such that the error e reflects the mismatch between the system G and the feedforward controller F.

Remark 7 The value of $M(p)$ in (19) is piecewise constant, i.e., in each interval where $M(p)$ is constant the system behaves as a linear spring with spring constant $M(p)$.

This observation allows one to design the desired trajectory in a similar fashion as described in Section II, where a piecewise constant reference trajectory is exploited to tune the feedforward controller for a linear spring. Applying a step will lead to $\lim_{t\to\infty} e(t) = \gamma \left(1 - \frac{M(p)}{\sum_{j=1}^{N} w_j m_j(p_{ff})}\right)$, for some constant $\gamma \in \mathbb{R}$, hence the error reflects the mismatch between $M(p)$ and $\sum_{j=1}^{N} w_j m_j(p_{ff})$. To find the N unknown weights w_j, a desired trajectory should consist of at least N different amplitudes to ensure that each weighting $w_j, j \in \{1, ..., N\}$ is visible in the error. A possible choice to achieve this is given in Fig. 7, where the desired trajectory is piecewise constant with N different amplitudes such that in each section an extra play operator is active, i.e., at the k^{th} amplitude the stiffness is given by $M(p) = \sum_{j=1}^{k} w_j$.

This results in the following systematic procedure to manually tune the feedforward controller for hysteresis, Contribution C3.

Procedure 2 (Feedforward Tuning for Hysteresis)

1) Select N play operators and their corresponding thresholds r_j, $j \in \{1, ..., N\}$.
2) Choose a piecewise constant desired trajectory with at least N different amplitudes, see, e.g., the trajectory in Fig. 7. The amplitudes should be in increasing order to ensure only one new play operator becomes active with each amplitude change.
3) Start with the first weighting $w_1 \gg 0$, i.e., no feedforward. This corresponds to the part of the reference with the lowest amplitude. Gradually decrease w_1. Choose the length of each step sufficiently long to eliminate transients.
4) The optimal feedforward parameter w_1 is given by the value of w_1 for which the error was minimal in the section of the reference where only the first play operator is active.
5) Use this approach to tune the parameters $w_2, ..., w_N$ in increasing order, starting from $w_j >> 0$ $j \in \{1, ..., N\}$. To tune the weight w_j evaluate the error in the section where $|p_i| > r_i$, for all $i \in \{1, ..., j\}$, and $|p_k| < r_k$ for all $k \in \{j+1, ..., N\}$.
6) If the performance is insufficient, increase the number of play operators N, adjust the threshold values r_j, $j \in \{1, ..., N\}$, and restart the procedure.

VI. Experimental validation

In this section Procedure 2 is applied to a piezoelectric actuator to determine a feedforward controller that compensates its hysteretic behaviour.

A. Implementation aspects

Several implementation aspects need to be considered.

- The feedforward controller is implemented digitally, typically using a sample and hold device. Hence, a discrete-time variant of the feedforward controller (22) should be determined. In this section a forward Euler discretization scheme is exploited.
- The initial conditions $p_{mff}(0)$ and $u(0)$ should be chosen carefully, to match the initial state of the piezoelectric actuator.

B. Validation

In Fig. 7 a desired trajectory and its corresponding error during execution in Procedure 2 is given. In this experiment the number of play operators is four, i.e., $N = 4$, and their corresponding threshold values are chosen equidistant over

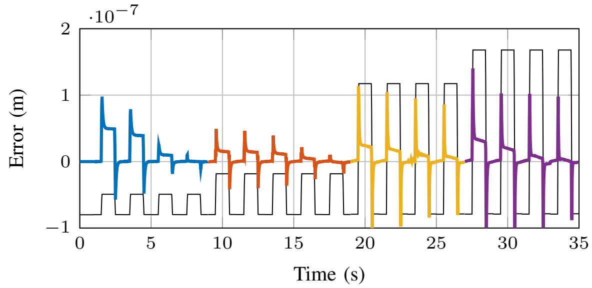

Fig. 7. Experimental validation of the tuning steps as introduced in Procedure 2 applied to a piezoelectric actuator. The desired trajectory (scaled and shifted in black (—)) is piecewise constant with 4 different amplitudes, each amplitude ensures an additional momentum p_j exeeds its threshold. The four tuning steps: decreasing w_1 in blue (—); decreasing w_2 in red (—); decreasing w_3 in yellow (—); decreasing w_4 in purple (—).

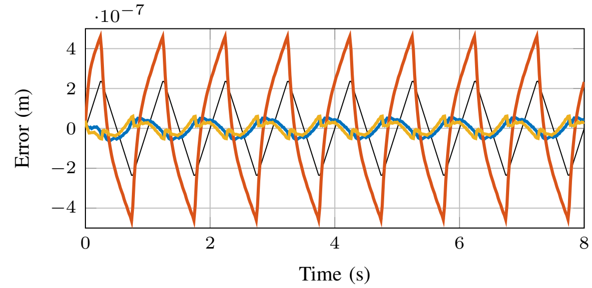

Fig. 8. Closed-loop experiment with piezoelectric actuator with a triangular desired trajectory (scaled and shifted in black (—)). Three different cases are studied: only feedback (—); with hysteresis compensation and corresponding identification procedure of 4 weights as in [9] (—); with feedforward compensation determined using the control-relevant identification procedure of four weights as described in Procedure 2 (—).

the input range from 0 to 250, i.e., $r_0 = 0$, $r_1 = 62.5$, $r_2 = 125$, $r_3 = 187.5$.

To validate the the performance of the feedforward controller the error signal during a closed-loop experiment with a triangular desired trajectory as depicted in Fig. 8. Clearly the cases with both feedback and feedforward outperform the case where only a feedback controller is active. Moroever, in terms of the maximum error the feedforward controller identified using the control-relevant identification Procedure 2 outperforms the feedforward controller determined independent of the controller [9].

VII. Conclusions

This paper presents a direct tuning approach for hysteresis that alleviates the need for forward modelling of hysteresis. The closed-loop tuning approach leads to a control-relevant feedforward controller by design. This is achieved by rewritting the Prandtl-Ishlinskii model as a hybrid-MEM-element. The favourable property of a hybrid-MEM-element, having a unique inverse, directly leads to a feedforward controller to compensate for the hysteresis phenomena. It is shown that each of the parameters in the feedforward controller can be distinguished in the error when performing closed-loop experiments with an appropriate desired trajectory. This leads to a systematic manual tuning approach that consists out of a series of closed-loop experiments during which each of the parameters are gradually changed to minimize the error. This results in a feedforward controller that is relevant for closed-loop control and without any intensive identification procedure.

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