Technische Universiteit
Eindhoven
University of Technology

On the Properties of Iterative Schemes

Tom Oomen, Cristian R. Rojas, Håkan Hjalmarsson, Bo Wahlberg

Tom Oomen
Department of Mechanical Engineering
Eindhoven University of Technology

Cristian R. Rojas
Håkan Hjalmarsson
Bo Wahlberg
School of Electrical Engineering
KTH - Royal Institute of Technology

Background

The idea of using iterative experiments appears in many system identification approaches. In this research, the value of iterations and the limits of accuracy are investigated.

ℓ_2-Induced Gain Estimation

Case study: gain estimation [1]

- perform iterative experiments on G, see Fig. 1 (left)
- corresponding transfer function: Fig. 1 (right)
- resulting u_k for $k \to \infty$:
  sinusoid with frequency $\omega^\star = \arg\sup_\omega |G(\omega)|$
- result: $\lim_{k\to\infty} \hat{\gamma}_k = \lim_{k\to\infty} \frac{\|y_k\|_2}{\|u_k\|_2} = \|G\|_\infty$
- resembles power iterations method

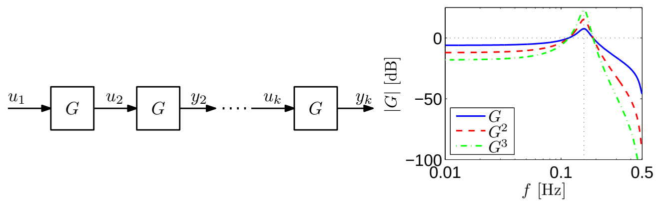

Figure 1: Simplified setup.

Extended setup

A more realistic setup is considered in Fig. 2, including

- noise: e_k is assumed ZMWN with variance λ_e
- normalization: α_k due to bound on $\|u_k\|$

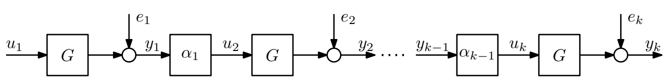

Figure 2: Considered setup.

Analysis

Resulting Spectrum

The extended setup in Fig. 2 is investigated through a spectral analysis. Assuming convergence for $k \to \infty$, then

$$\Phi_{u_\infty}(\omega) = \frac{\lambda_e}{\left(\frac{1}{2\pi}\int_{2\pi} |G(\omega)|^2 \Phi_{u_\infty}(\omega) d\omega + \lambda_e\right) - |G(\omega)|^2}$$

Observations:

- $\Phi_{u_\infty}(\omega)$ has maximum at $\omega^\star$
- $\Phi_{u_\infty}(\omega) \geq \frac{\lambda_e}{\alpha_\infty}$

Convergence Analysis

Extended system satisfying eigenvalue equation

$$\frac{1}{\alpha_\infty^2}\begin{bmatrix}\Phi_{u_\infty}(\omega)\\ 1\end{bmatrix} = \begin{bmatrix} |G|^2\Phi_{u_\infty}(\omega) & \lambda_e \\ \frac{1}{\pi}\int_\pi |G(\omega)|^2 \Phi_{u_\infty}(\omega) d\omega & \lambda_e \end{bmatrix}\begin{bmatrix}\Phi_{u_\infty}(\omega)\\ 1\end{bmatrix} \quad (1)$$

- convergence proof via Hilbert projective metric [2]
- ω-discretization: computation of $\Phi_{u_\infty}(\omega)$ for given G

Example

Given G in Fig. 1, (1) gives $\Phi_{u_\infty}(\omega)$ in Fig. 3.

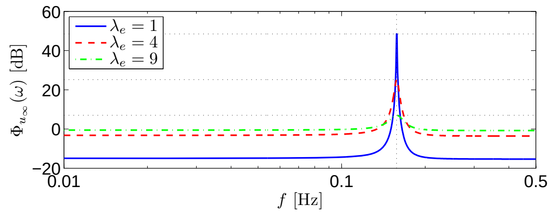

Figure 3: Limit input spectrum.

Implications

Bias Analysis for Non-Parametric Estimation

The nonparametric estimator in [1] can be written as

$$\hat{\gamma}_k = \sqrt{\frac{\frac{1}{2\pi}\int_{2\pi} |G(\omega)|^2 \Phi_{u_{k-1}}(\omega) d\omega}{\frac{1}{2\pi}\int_{2\pi} \Phi_{u_{k-1}}(\omega) d\omega}}$$

Combining this with the limit spectrum Φ_{u_∞} reveals

- $\hat{\gamma}_\infty = \|G\|_\infty$ for $\lambda_e = 0$
- $\hat{\gamma}_\infty < \|G\|_\infty$ for $\lambda_e > 0$ (biased)

Limits of Accuracy

Fisher information matrix

$$I_\theta = \sum_{l=1}^{k} \frac{1}{2\pi\lambda_e} \int_{-\pi}^{\pi} G'(e^{j\omega}, \theta) \Phi_{u_l}(\omega) \left(G'(e^{-j\omega}, \theta)\right)^T d\omega$$

- additivity property
  $\Rightarrow$ only increase of information for increasing k
- if $\Psi_{u_l}(\omega) = \delta(\omega - \omega^*)$
  $\Rightarrow$ optimal accuracy for $G(\omega^\star)$ for FIR model [3]

Final Remarks

Analysis of iterative experiments in identification

- case study: non-parametric ℓ_2-gain estimation

Present extensions

- finite time implementation: time reversal
  - only one experiment per iteration
- MIMO: multiple experiments per iteration
- nonparametric Hankel-norm estimation

Future work: analysis of the value of iterations in

- iterative learning control
- iterative identification and control
- iterative feedback tuning

References

[1] B. Wahlberg, M. Barenthin Syberg, and H. Hjalmarsson. Non-parametric methods for $\mathcal{L}_2$-gain estimation using iterative experiments. *Automatica*, 46(8):1376–1381, 2010.
[2] C.R. Rojas, T. Oomen, H. Hjalmarsson, and B. Wahlberg. *In preparation.*
[3] B. Wahlberg, H. Hjalmarsson, and P. Stoica. On Optimal Input Signal Design for Frequency Response Estimation. *Proc. 49th Conf. Dec. Contr.*, 2010.