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Robust-Control-Relevant Coprime Factor Identification with Application to Model Validation of a Wafer Stage ⋆

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Abstract: The performance of robust controllers depends on the set of candidate plants, but at present this intimate connection is untransparent. The aim of this paper is to construct a model set to improve the performance in a subsequent robust control design. Analysis of uncertainty structures reveals that there is an unexploited freedom in the realization of coprime factorizations in the dual-Youla uncertainty structure. The main result of this paper is a specific coprime factorization that results in model sets that are tuned for robust control. The presented coprime factorization can be identified directly from data. Application of the proposed methodology to an industrial wafer stage reveals improved model validation results.

1. INTRODUCTION

The purpose of any model should be taken into account when evaluating its quality. In case the goal of the model is subsequent control design, the model merely needs to represent the phenomena that are relevant for control. This is especially true for flexible mechanical systems, where the identification of full-order models is often illusive due to the occurrence of parasitic dynamics. The observation that models should be tuned towards their goal has led to the development of iterative identification and control design schemes [Schrama, 1992], [Gevers, 1993].

Iterative identification and control design schemes that are based on nominal models resulted in different outcomes. In fact, discrepancies between the true plant and the model can lead to a lack of convergence [Hjalmarsson, 2005], requiring cautious controller updates.

A robust control design [Zhou et al., 1996] ensures that a designed controller achieves satisfactory performance when implemented on the true plant without requiring cautious controller updates. The quality of the designed robust controller depends on the entire model set instead of solely on the nominal model. In Gevers et al. [2003], model sets based on the prediction error identification are investigated in a control-relevant setting. However, the approach relies on full-order models. In contrast, in the present paper a framework is investigated that enables the use of reduced-order models.

In de Callafon and Van den Hof [1997], a frequency domain approach to identifying control-relevant model sets is presented. Frequency domain approaches are especially useful for reliably identifying multivariable flexible mechanical systems. Specifically, in de Callafon and Van den Hof [1997], the dual-Youla uncertainty structure [Douma and Van den Hof, 2005] is used to ensure control-relevance in iterative identification and robust control design. Although their approach results in robust-control-relevant model sets, the uncertainty is determined in a weighted domain. This essentially limits the application to multiple scalar perturbations. In the present paper, it is shown that there is an unexplored freedom in the coprime factorizations that are used to construct the dual-Youla uncertainty structure. Indeed, among the class of admissible factorizations, certain realizations have control-relevant properties.

The main contribution of the present paper is the construction of a specific coprime factor realization that characterizes the uncertainty in a domain that is directly interpretable in terms of a robust control criterion. These coprime factors are identified directly from data in a control-relevant sense and are used in conjunction with a dual-Youla uncertainty structure. The proposed methodology thus fully exploits the freedom in the realizations, which is an advantage over previously published results. As a result, the proposed framework directly generalizes and extends previous results, including de Callafon and Van den Hof [1997], to multivariable uncertainty blocks.

The paper is organized as follows. In Section 2, the robust-control-relevant identification problem is stated. In Section 3, a novel coprime factorization identification procedure is presented. In Section 4, specific coprime factor realizations are introduced, leading to a robust-control-relevant uncertainty structure. In Section 5, a validation-based uncertainty modeling approach is briefly introduced. Finally, experimental results of a wafer stage system are presented in Section 6, followed by conclusions in Section 7.

2. PROBLEM FORMULATION

In this section, the robust-control-relevant identification problem is formulated and relevant background material is presented.

2.1 Control goal

The considered criterion is given by

$$J(P,C) = \|WT(P,C)V\|_\infty, \tag{1}$$

where $W = \operatorname{diag}(W_y, W_u)$ and $V = \operatorname{diag}(V_2, V_1)$ are bistable weighting filters. These weighting filters are for

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instance used to specify the desired shape of the closed-loop transfer functions. Bistability of the weighting filters is nonrestrictive, since weighting filters can be absorbed in the plant P. In (1), $T(P,C)$ is given by

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} P \\ I \end{bmatrix} (I+CP)^{-1} [C \ I] \begin{bmatrix} r_2 \\ r_1 \end{bmatrix} = T(P,C) \begin{bmatrix} r_2 \\ r_1 \end{bmatrix}, \tag{2}$$

see Figure 1. The goal is to determine

$$C^{\text{opt}} = \arg\min_C J(P_o, C), \tag{3}$$

where C^{opt} is the optimal controller and P_o the true plant.

2.2 Nominal model identification

To perform the actual optimization in (3), the knowledge regarding P_o is required that is reflected by a model $\hat{P}$. The performance of a controller based on $\hat{P}$ may not be optimal when applied to P_o. In fact, it is bounded by

$$J(P_o, C) \leq J(\hat{P}, C) + \|W(T(P_o, C) - T(\hat{P}, C))V\|_\infty. \tag{4}$$

The main conclusion of (4) is that the plant model $\hat{P}$ should be accurate for its subsequent control goal. The triangle inequality (4) is at the heart of many iterative identification and control design approaches and results in the following control-relevant identification criterion [Schrama, 1992].

Definition 1. The control-relevant identification criterion is defined as

$$\min_{\hat{P}} \|W(T(P_o, C^{\text{exp}}) - T(\hat{P}, C^{\text{exp}}))V\|_\infty. \tag{5}$$

In (5), C^{exp} denotes the controller that is present during the identification experiment, hence (5) is a closed-loop identification problem. The superscript exp is omitted if no confusion can arise.

2.3 Towards robust-control-relevant model sets

No realistic system can be described exactly by a nominal model. To guarantee that the designed controller also performs well with the true system, the nominal model can be equipped with an uncertainty model. In this case, the uncertain plant contains several candidate plants, presumably containing the true plant P_o.

In this paper, the set of plants is constructed by first identifying a nominal plant model, see Section 2.2, followed by a validation-based uncertainty modeling approach. Similar to the identification of nominal plant models, the uncertain plant set should be tuned towards its goal, leading to a robust-control-relevant model set.

To enable a direct use of the uncertain model set in robust control based on $\mathcal{H}_\infty$-optimization [Zhou et al., 1996], the uncertain model set is of the form

$$\mathcal{P} = \left\{ P \middle| P = \mathcal{F}_u(H, \Delta), \|\Delta\|_\infty < \gamma \right\}, \tag{6}$$

where Δ can be subject to additional structural requirements, H contains $\hat{P}$ and the uncertainty structure, and $\mathcal{F}_u$ denotes the upper linear fractional transformation. Associated with $\mathcal{P}$ is the worst-case performance

$$J_{\text{WC}}(\mathcal{P}, C) = \sup_{P \in \mathcal{P}} J(P, C). \tag{7}$$

Clearly, if $P_o \in \mathcal{P}$, then the bound

$$J(P_o, C) \leq J_{\text{WC}}(\mathcal{P}, C) \tag{8}$$

applies.

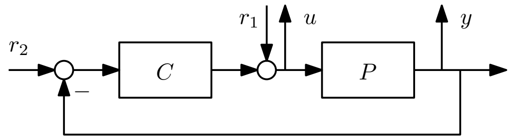

Fig. 1. Feedback configuration.

In view of (7), a robust-control-relevant model set is a model set that results in a small value of J_{WC}. In analogy to the iterative identification and control approach discussed in Section 2.2, the upper bound (8) can be tightened by alternating between the identification of a robust-control-relevant model set $\mathcal{P}$ and the design of a robust controller C. Specifically, following de Callafon and Van den Hof [1997], at iteration step k,

(1) Determine a model set $\mathcal{P}$ such that

$$\mathcal{P}^{<k+1>} = \arg\min_{\mathcal{P}} J_{WC}(\mathcal{P}, C^{<k>}) \tag{9}$$

$$\text{subject to } P_o \in \mathcal{P}. \tag{10}$$

(2) Perform a model-based robust controller design

$$C^{<k+1>} = \arg\min_C J_{WC}(\mathcal{P}^{<k+1>}, C). \tag{11}$$

Clearly, this iterative approach is monotonically convergent in J_{WC}. The estimation of a robust-control-relevant model set, which is the main problem that is addressed in this paper, is given by (9) and (10).

3. NOMINAL CONTROL-RELEVANT COPRIME FACTOR IDENTIFICATION

In this section, the control-relevant identification criterion (5) is recast as a coprime factor identification problem. Note that the internal structure of the model does not affect the criterion value in (5), hence any factorization of the plant is equal in the sense of the control relevant criterion. The specific purpose of the identified factorization becomes clear in Section 4 when the nominal model is equipped with an uncertainty model.

3.1 Robust-control-relevant coprime factors

The following result is required in the subsequent derivations. For the definition of a Right Coprime Factorization (RCF) and a Left Coprime Factorization (LCF), see, e.g., Zhou et al. [1996].

Theorem 2. (LCF with co-inner numerator). Let $G(z) \in \mathcal{R}^{n_z \times n_w}$, $n_w \geq n_z$ with minimal state-space realization (A, B, C, D), assuming D has full row rank. Then, there exists an LCF of G, $G = \tilde{D}^{-1}\tilde{N}$ such that $\tilde{N}$ is co-inner, i.e., $\tilde{N}\tilde{N}^* = I$, if and only if $G(e^{j\omega})G^{\sim}(e^{j\omega}) > 0 \ \forall \omega \in [0, 2\pi]$.

See Chu [1988] for a proof of Theorem 2 and state-space formulas for the computation of an LCF with co-inner numerator. The following proposition constitutes the main result of this section.

Proposition 3. Consider the control-relevant identification criterion (5) and let $\{\tilde{N}_e, \tilde{D}_e\}$ be an LCF of $[CV_2 \ V_1]$ with co-inner numerator, where $\tilde{N}_e = \left[\tilde{N}_{e,2} \ \tilde{N}_{e,1}\right]$. Then, (5) is equivalent to

$$\min_{\hat{N}, \hat{D}} \|W(\begin{bmatrix} N_o \\ D_o \end{bmatrix} - \begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix})\|_\infty, \tag{12}$$

where

$$\begin{bmatrix} N_o \\ D_o \end{bmatrix} = \begin{bmatrix} P_o \\ I \end{bmatrix} (\tilde{D}_e + \tilde{N}_{e,2} V_2^{-1} P_o)^{-1} \quad (13)$$

$$\begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} \hat{P} \\ I \end{bmatrix} (\tilde{D}_e + \tilde{N}_{e,2} V_2^{-1} \hat{P})^{-1}. \quad (14)$$

A proof of Proposition 3 follows along similar lines as in Oomen and Bosgra [2008a]. The novelty of Proposition 3 is that, in contrast to Oomen and Bosgra [2008a], the result of Theorem 2 does not lead to excessive orders of the coprime factorizations $\{N_o, D_o\}$, see (13), and $\{\hat{N}, \hat{D}\}$, see (14). Selecting an LCF such that $\tilde{N}_e$ is co-inner is essential to guarantee preservation of the $\mathcal{H}_\infty$-norm in (12).

The following results reveal that $\{N_o, D_o\}$ and $\{\hat{N}, \hat{D}\}$ are coprime factorizations of P_o and $\hat{P}$, respectively.

Proposition 4. Let $\tilde{N}_{e,2}$ and $\tilde{N}_{e,1}$ be defined as in Proposition 3. Then, $\tilde{N}_{e,2}$ and $\tilde{N}_{e,1}$ are left coprime if and only if $[CV_2 \ V_1]$ has no RHP transmission zeros.

A proof is omitted due to space limitations. Since it is assumed that $V_1^{-1} \in \mathcal{RH}_\infty$, $\tilde{N}_{e,2}$ and $\tilde{N}_{e,1}$ are right coprime by virtue of Proposition 4. This enables the following result.

Proposition 5. Let $T(P, C) \in \mathcal{RH}_\infty$ and let $\{\tilde{N}_e, \tilde{D}_e\}$ be an LCF of $[CV_2 \ V_1]$. Then, $\{P(\tilde{D}_e + \tilde{N}_{e,2}V_2^{-1}P)^{-1}, (\tilde{D}_e + \tilde{N}_{e,2}V_2^{-1}P)^{-1}\}$ is an RCF of P, where $\tilde{N}_e = \begin{bmatrix}\tilde{N}_{e,2} & \tilde{N}_{e,1}\end{bmatrix}$.

A proof of (5) follows along similar lines as in Oomen and Bosgra [2008a], the main difference is that the result of Proposition 4 is employed.

Proposition 3 and 5 reveal that the control-relevant identification criterion (5) can be recast as a coprime factor identification problem. In Section 4, it is shown that the specific coprime factor realization results in a robust-control-relevant uncertainty structure. In contrast, in Oomen and Bosgra [2008a] it is shown that normalized coprime factorizations generally do not directly lead to robust-control-relevant model sets in the sense of (9) and (10).

3.2 A frequency domain identification problem

The optimization problem in (12) is not straightforward due to the presence of the $\mathcal{H}_\infty$-norm. The main idea is to use the frequency domain interpretation of the $\mathcal{H}_\infty$-norm to obtain a tractable optimization problem, i.e.,

$$\|G\|_\infty = \sup_{\omega \in [0, 2\pi)} \bar{\sigma}\left(G(e^{j\omega})\right). \quad (15)$$

The key issue is that in (5) and hence (12), knowledge regarding P_o is required. This knowledge is obtained by means of a frequency domain identification procedure for $\omega_i \in \Omega$. Generally, Ω is a discrete frequency grid, resulting from a finite time identification experiment. In contrast, the $\mathcal{H}_\infty$-norm, see (15), involves an optimization over a continuous frequency grid.

Minimization of the $\mathcal{H}_\infty$-norm using a discrete frequency grid can be performed in case suitable prior assumptions are imposed, see Chen and Gu [2000]. However, in this case the resulting uncertainty hinges on the validity of these assumptions. This has led to overly conservative results, especially for flexible mechanical systems, see, e.g., Friedman and Khargonekar [1995]. In contrast, in this paper a lower bound of the $\mathcal{H}_\infty$-norm is considered, where θ is an appropriate parameterization for $\{\hat{N}, \hat{D}\}$, i.e., $\begin{bmatrix}\hat{N}^T(\theta) & \hat{D}^T(\theta)\end{bmatrix}^T$. The behavior in between the frequency grid is addressed in the subsequent model validation step, see Section 5.

Proposition 6. The optimization problem

$$\min_{\theta} \max_{\omega_i \in \Omega} \bar{\sigma}\big(W\big(\begin{bmatrix} N_o(\omega_i) \\ D_o(\omega_i) \end{bmatrix} - \begin{bmatrix} \hat{N}(\theta, \omega_i) \\ \hat{D}(\theta, \omega_i) \end{bmatrix}\big)\big) \quad (16)$$
$$\text{subject to } T(\hat{P}, C) \in \mathcal{RH}_\infty.$$

minimizes a lower bound of the $\mathcal{H}_\infty$-norm in the optimization problem (12).

Proof. Follows directly from (15).

In Proposition 6, $\{N_o(\omega_i), D_o(\omega_i)\}$ can be determined directly from data if $T(P_o, C^{\exp})$ is identified for $\omega_i \in \Omega$. The identification of $T(P_o, C^{\exp})$ is in fact an open-loop type identification problem. Subsequently, $\{N_o(\omega_i), D_o(\omega_i)\}$ is obtained if $T(P_o(\omega_i), C^{\exp}(\omega_i))$ is appended with weighting filters and multiplied to the right by $\begin{bmatrix}\tilde{N}_{e,2}(\omega_i) & \tilde{N}_{e,1}(\omega_i)\end{bmatrix}^*$.

3.3 Optimization procedure

In this section, the optimization problem (16) is investigated in further detail. A tailor-made parametrization is employed, i.e., $\{\hat{N}, \hat{D}\}$ are parameterized as

$$\begin{bmatrix} \hat{N}(\theta) \\ \hat{D}(\theta) \end{bmatrix} = \begin{bmatrix} B(\theta) \\ A(\theta) \end{bmatrix} (\tilde{D}_e A(\theta) + \tilde{N}_{e,2} V_2^{-1} B(\theta))^{-1}, \quad (17)$$

where $A(\theta) \in \mathbb{R}[\xi]^{n_w \times n_w}$, $B(\theta) \in \mathbb{R}[\xi]^{n_z \times n_w}$ are polynomial matrix fraction descriptions that are linear in the parameters. In case of (17), the following result holds.

Proposition 7. Consider the factorization (17). Then, the following statements are equivalent:

(1) $T(\hat{P}(\theta), C) \in \mathcal{RH}_\infty$.
(2) $\begin{bmatrix} B(\theta) \\ A(\theta) \end{bmatrix} (\tilde{D}_e A(\theta) + \tilde{N}_{e,2} V_2^{-1} B(\theta))^{-1} \in \mathcal{RH}_\infty$

The proof follows along similar lines as in Oomen and Bosgra [2008a]. Proposition 7 reveals that the main purpose of the tailor-made parameterization is to connect the requirement of $\hat{P}$ being stabilized by C, which is a necessary condition for control relevance, see (16), to the fact that $\{\hat{N}, \hat{D}\}$ are stable, which is a necessary condition for coprimeness over $\mathcal{H}_\infty$.

Typically, criterion (16) together with parametrization (17) constitute an optimization problem that is nonlinear in the parameters and nonsmooth. Hence, no analytic solution is available and efficient gradient-based optimization algorithms cannot be used directly. Therefore, an iterative method is considered, see also Oomen and Bosgra [2008a]. Specifically, Lawson's algorithm [Rice, 1964] is used that iteratively solves nonlinear least squares problems to approximate the $\mathcal{H}_\infty$-norm. SK-iterations [Sanathanan and Koerner, 1963] are used to generate good initial estimates for subsequent Gauss-Newton iterators, which ensures convergence to a local minimum.

The solution approach is not necessarily convergent. A numerically reliable approach, which is an extension of the approach in Bultheel et al. [2005], is used to diminish numerical problems. Extensive experiments reveal promising results. Generally, sensibly selected weighting filters W and V result in $T(\hat{P}, C) \in \mathcal{RH}_\infty$, see Proposition 7. In contrast to identifying normalized coprime factorizations [Van den Hof et al., 1995], the presented approach does not require iterations over coprime factor realizations, hence an additional nonconvergent iteration is avoided.

4. ROBUST-CONTROL-RELEVANT MODEL SETS

4.1 Dual-Youla perturbation structure

In this section, an uncertainty model structure is presented that is suitable for robust-control-relevant model sets in the sense of (9). Recall that a validation-based uncertainty modeling approach is being pursued, where first a nominal coprime factor model $\hat{P} = \hat{N}\hat{D}^{-1}$ is estimated, followed by an estimation of the size of the uncertainty to ensure (10). Two requirements can be distinguished. Firstly, the true system should be in the model set for some norm-bounded and hence stable Δ, see (10) and (6). Secondly, the model set should be control-relevant, i.e., the worst case performance in (9) should decrease. The observation that both P_o and $\hat{P}$ are stabilized by C in a control-relevant setting, see (5), enables the following result that satisfies both requirements.

Theorem 8. Let $\hat{P}$ be internally stabilized by the negative feedback controller C and let $\{\hat{N}, \hat{D}\}$ and $\{N_c, D_c\}$ be an RCF over $\mathcal{RH}_\infty$ of $\hat{P}$ and C, respectively. Then, all plants P stabilized by C are given by

$$P = (\hat{N} + D_c\Delta_u)(\hat{D} - N_c\Delta_u)^{-1}, \ \Delta_u \in \mathcal{RH}_\infty \tag{18}$$

The proof is dual to the Youla-Kučera parametrization, see, e.g., Zhou et al. [1996]. The dual-Youla parametrization is a refinement of coprime factor uncertainty. Specifically, only candidate plants are included in $\mathcal{P}$ that are actually stabilized by C. This leads to the most parsimonious set $\mathcal{P}$, see also Douma and Van den Hof [2005] for further details.

Coprime factorizations of $\hat{P}$ are discussed in Section 3. In the next section, a specific choice of coprime factorizations of C is presented. In Section 4.3, it is shown that the presented choice leads to a robust-control-relevant perturbation model in the sense that the size of the uncertainty model directly affects the robust control criterion, i.e., no weighting filters are involved.

4.2 (W_u, W_y)-normalized coprime factorizations

In this section, a coprime factorization of C is presented that is (W_u, W_y)-normalized. This result is required to state the main result in Section 4.3, which is to select a control-relevant coordinate frame of the dual-Youla model uncertainty structure. (W_u, W_y)-normalization is defined as follows.

Definition 9. The pair $\{N_c, D_c\}$ is an (W_u, W_y)-normalized RCF of C if it is an RCF of C and in addition,

$$\begin{bmatrix} W_u N_c \\ W_y D_c \end{bmatrix}^* \begin{bmatrix} W_u N_c \\ W_y D_c \end{bmatrix} = I. \tag{19}$$

The following proposition enables a state-space computation of (W_u, W_y)-normalized coprime factorizations.

Proposition 10. Given W_u, W_y, C with minimal state-space realizations (A_u, B_u, C_u, D_u), $(A_{yi}, B_{yi}, C_{yi}, D_{yi})$, and (A_c, B_c, C_c, D_c), of W_u, W_y^{-1}, and C, respectively. Then, a (W_u, W_y)-normalized RCF $\{N_c, D_c\}$ of C is given by

$$\begin{bmatrix} D_c \\ N_c \end{bmatrix} = \left[\begin{array}{c|c} \mathcal{A} + \mathcal{B}F & \mathcal{B}R^{-\frac{1}{2}} \\ \hline \mathcal{C} + \mathcal{D}F & \mathcal{D}R^{-\frac{1}{2}} \end{array}\right], \tag{20}$$

where

$$Z = R + \mathcal{B}^* X \mathcal{B}, \quad F = -Z^{-1}(\mathcal{B}^* X \mathcal{A} + \mathcal{D}^* \mathcal{C}),$$

X is the unique, stabilizing solution to the DARE

$$\tilde{\mathcal{A}}^* X \tilde{\mathcal{A}} - X - \tilde{\mathcal{A}}^* X \mathcal{B}(\mathcal{B}^* X \mathcal{B} + R)^{-1} \mathcal{B}^* X \tilde{\mathcal{A}} + Q = 0,$$

and

$$\tilde{\mathcal{A}} = \mathcal{A} - \mathcal{B}R^{-1}\mathcal{D}^*\mathcal{C}, \qquad Q = \mathcal{C}^* \tilde{R}^{-1} \mathcal{C}$$

$$R = I + \mathcal{D}^*\mathcal{D} > 0, \qquad \tilde{R} = I + \mathcal{D}\mathcal{D}^* > 0$$

$$\mathcal{A} = \begin{bmatrix} A_u & B_u C_c & B_u D_c C_{yi} \\ 0 & A_c & B_c C_{yi} \\ 0 & 0 & A_{yi} \end{bmatrix}, \qquad \mathcal{B} = \begin{bmatrix} B_u D_c D_{yi} \\ B_c D_{yi} \\ B_{yi} \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} 0 & 0 & C_{yi} \\ 0 & C_c & D_c C_{yi} \end{bmatrix}, \qquad \mathcal{D} = \begin{bmatrix} D_{yi} \\ D_c D_{yi} \end{bmatrix}.$$

Proof. Define using (20) the state-space realization

$$\begin{bmatrix} D_w \\ N_w \end{bmatrix} = \left[\begin{array}{c|c} \mathcal{A} + \mathcal{B}F & \mathcal{B}Z^{-\frac{1}{2}} \\ \hline F & Z^{-\frac{1}{2}} \\ \begin{bmatrix} C_u & D_u C_c & D_u D_c C_{yi} \end{bmatrix} + D_u D_c D_{yi} F & D_u D_c D_{yi} Z^{-\frac{1}{2}} \end{array}\right],$$

where $\{N_w, D_w\}$ is in fact a normalized RCF of $W_u C W_y^{-1}$, e.g., [Zhou et al., 1996, Theorem 21.25]. If $\{N_w, D_w\}$ is a normalized RCF, then $\{N_c, D_c\} = \{W_u^{-1} N_w, W_y^{-1} D_w\}$ is an RCF of C since $W_u, W_u^{-1}, W_y, W_y^{-1} \in \mathcal{RH}_\infty$. In addition, in this case (9) holds, which completes the proof that $\{N_c, D_c\}$ is (W_u, W_y)-normalized.

In the next section, the role of (W_u, W_y)-normalization in the dual-Youla parameterization is discussed.

4.3 A robust-control-relevant dual-Youla parameterization

In this section, the plant RCF $\{\hat{N}, \hat{D}\}$, see Section 3, and controller RCF $\{N_c, N_c\}$, see Section 4.2, are used to construct a robust-control-relevant perturbation structure. Propositions 11 and 12 constitute the main result of this section. The performance of a candidate model, see (6), is given by the following proposition.

Proposition 11. Given a candidate plant model $P \in \mathcal{P}$, corresponding to a certain Δ, see (6). Then, the performance of this P is given by

$$J(P, C) = \|\mathcal{F}_u(\hat{M}, \Delta)\|_\infty, \tag{21}$$

where

$$\hat{M} = \left[\begin{array}{c|c} 0 & \begin{bmatrix} \tilde{N}_{e,2} & \tilde{N}_{e,1} \end{bmatrix} \\ \hline \begin{bmatrix} W_y D_c \\ -W_u N_c \end{bmatrix} & W \begin{bmatrix} \hat{P} \\ I \end{bmatrix} (I + C\hat{P})^{-1} \begin{bmatrix} C & I \end{bmatrix} V \end{array}\right]. \tag{22}$$

Proof. $\hat{M}_{22}$ follows directly since this is the nominal performance $WT(\hat{P}, C)V$. In addition, $\hat{M}_{11}$ and $\hat{M}_{21}$ follow directly if the dual-Youla structure (18) is considered in feedback with C. Regarding $\hat{M}_{12}$,

$$\hat{M}_{12} = (\hat{D} + C\hat{N})^{-1} \begin{bmatrix} C & I \end{bmatrix} V \tag{23}$$

$$= \hat{D}^{-1}(\tilde{D}_e + \tilde{N}_{e,2} V_2^{-1} \hat{P})^{-1} \tilde{N}_e = \begin{bmatrix} \tilde{N}_{e,2} & \tilde{N}_{e,1} \end{bmatrix}, \tag{24}$$

where the latter equality follows from (14).

The following proposition reveals robust-control-relevance of the uncertainty model structure.

Proposition 12. Consider P in a set defined by (18). Then,

$$J_{\mathrm{WC}} \leq \|\hat{M}_{22}\|_\infty + \|\Delta\|_\infty. \quad (25)$$

Proof. Observing that $\hat{M}_{21}$ and $\hat{M}_{12}$ are inner and co-inner, respectively, implies that $\exists \hat{M}_{21}^{\perp}, \hat{M}_{12}^{\perp}$ such that $\begin{bmatrix}\hat{M}_{21} & \hat{M}_{21}^{\perp}\end{bmatrix}, \begin{bmatrix}\hat{M}_{12} \\ \hat{M}_{12}^{\perp}\end{bmatrix} \in \mathcal{RH}_\infty$ are all-pass and

$$\mathcal{F}_u(\hat{M}, \Delta) = \hat{M}_{22} + \begin{bmatrix}\hat{M}_{21} & \hat{M}_{21}^{\perp}\end{bmatrix} \begin{bmatrix}\Delta & 0 \\ 0 & 0\end{bmatrix} \begin{bmatrix}\hat{M}_{12} \\ \hat{M}_{12}^{\perp}\end{bmatrix}, \quad (26)$$

Employing the triangle inequality and norm-preserving property of all-pass transfer function matrices proves (25).

Proposition 12 reveals that determining the minimum-norm of Δ such that (10) holds results in a robust-control-relevant model set. Specifically, the norm of Δ directly affects the control criterion J_{WC}. This is achieved by the specific coprime factor realization that is obtained in Sections 3 and 4.2. Due to the transparent interpretation in (25), the proposed approach can directly be applied to multivariable uncertainty blocks. In contrast, if different coprime factor realizations are chosen, then $\hat{M}_{21}$ and $\hat{M}_{12}$ are generally not norm-preserving. This results in a weighting of Δ in J_{WC} that clouds the interpretation.

5. MODEL VALIDATION

The size of the perturbation model, see (9), (25), is estimated using a validation-based uncertainty modeling approach, see Oomen and Bosgra [2008b, 2009]. The approach is an extension of Smith and Doyle [1992] and includes the following original contributions.

(1) a deterministic approach is pursued, using accurate, nonparametric, deterministic disturbance models.
(2) the approach is optimistic and does not increase the uncertainty if noisy data are used. This is especially useful if many validation experiments are performed. This is in sharp contrast to pessimistic approaches, see Chen and Gu [2000].
(3) averaging of the disturbances is enforced in a deterministic framework by means of an appropriate experiment design using periodic input signals.
(4) necessary and sufficient conditions for a frequency domain test are provided, enabling a computationally tractable model validation test for large data sets.

The result of the validation-based uncertainty modeling approach is both a nonparametric and a parametric bound for the minimum-norm validating Δ. Although the former bound is tighter, the latter is required in subsequent controller synthesis. This is further discussed in Section 6.3.

6. EXPERIMENTAL RESULTS

6.1 System description

In this section, the presented approach is applied to a prototype wafer stage system, see Figure 2. A wafer stage is a six degrees-of-freedom positioning system that is used in the production of integrated circuits. Although both the system and the results of the previous sections are multivariable, only one translational direction is considered to facilitate the presentation.

Fig. 2. Industrial wafer stage setup.

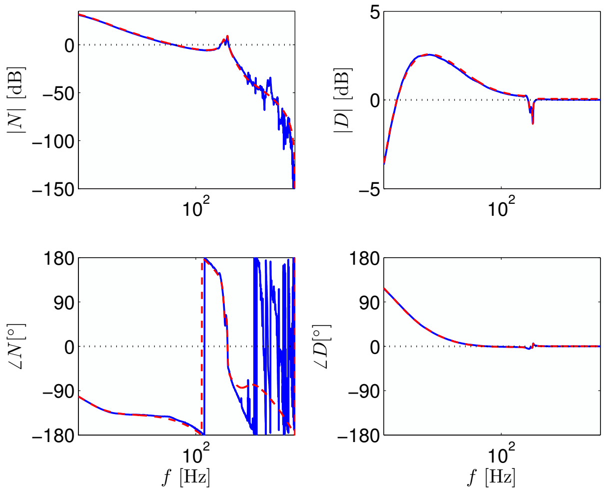

Fig. 3. Bode diagram of identified N_o, D_o (solid) and estimated $\hat{N}, \hat{D}$ (dashed).

6.2 Nominal model identification

Since the open-loop system is unstable, a stabilizing feedback controller C^{exp} with an approximate bandwidth of 10 Hz is present during the nominal identification. To increase performance, weighting filters are designed with a target bandwidth of 50 Hz. In Figure 3, the frequency responses of the true plant coprime factorizations N_o and D_o are depicted. In addition, the resulting nominal coprime factorizations $\hat{N}$, $\hat{D}$, minimizing (16), are depicted. Two observations are made. Firstly, the coprime factorizations are large in a control-relevant region, i.e., around the bandwidth. Secondly, this is precisely the region where the identified factors $\hat{N}$, $\hat{D}$ are accurately identified. As a consequence, the identified coprime factors are stable, see also Proposition 3.

6.3 Model validation

Using the nominal model, represented as the coprime factorization $\hat{N}$, $\hat{D}$, the dual-Youla uncertainty structure is constructed, see Section 4, and the model validation procedure, see Section 5, is applied. The resulting frequency-dependent upper bound for γ is depicted in Figure 4. Since the nonparametric bound cannot be used in robust controller synthesis, a static upper bound is determined.

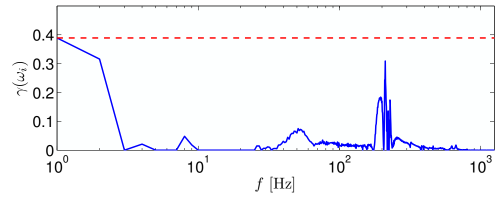

Fig. 4. Minimum-norm validating Δ in the FDMVOP (solid) and static overbound (dashed).

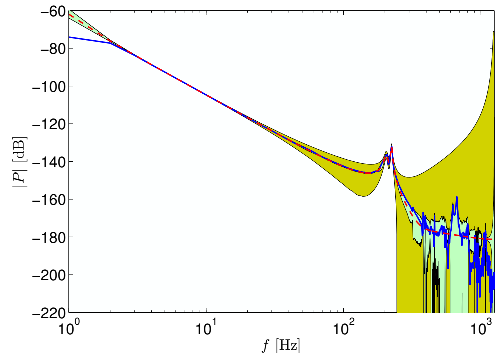

Fig. 5. Bode magnitude diagram of transfer function estimate of P_o (solid), model $\hat{P}$ (dashed), plant set $\mathcal{P}$ using nonparametric bound $\gamma(\omega_i)$ (cyan), plant set $\mathcal{P}$ using static overbound (yellow).

Although this bound is conservative, it is useful to illustrate control-relevance of the resulting model set. The corresponding identified plant sets are depicted in Figure 5. In addition, the estimated frequency response function of the open-loop plant P_o is depicted. Especially in case of the parametric overbound, control relevance of the uncertain model is confirmed by the results in Figure 5. Specifically, the uncertainty is small around the present bandwidth frequency of 10 Hz, which corresponds with the results in Section 6.2 and control design guidelines. Around 100 Hz, there is a slight increase in uncertainty. The model set is tight around the 180 Hz resonance, this resonance is indeed control relevant since it destabilizes the closed-loop system if not appropriately taken care of. Interestingly, all higher frequent resonances are very uncertain, hence these are indeed not control-relevant. Finally, note that the pursued approach attributes the nominal model deviation at 1 Hz to disturbances and not to model uncertainty. This uncertain model can be used directly in a robust control design, in particular in a skewed-μ synthesis.

7. CONCLUSIONS

In this paper, the unexplored freedom in coprime factor realizations in the dual-Youla uncertainty structure is exploited to develop a robust-control-relevant model set. In particular, the proposed coprime factor realizations can be identified directly from data in a control-relevant criterion. Subsequently, the validation-based uncertainty modeling approach is then performed in the same criterion. In this perspective, the paper provides a transparent connection between identification and model validation on the one hand, and robust control on the other hand. Application of the proposed approach to an industrial wafer stage has been shown to give improved model validation results.

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