Joint 48th IEEE Conference on Decision and Control and
28th Chinese Control Conference
Shanghai, P.R. China, December 16-18, 2009

Identification for Robust Inferential Control

Tom Oomen, Okko Bosgra, and Marc van de Wal

Abstract— **High measured performance does not imply that the true system performance is satisfactory. Indeed, in many systems, these performance variables cannot be measured directly and have to be inferred from the measured variables by using model knowledge. The aim of the present paper is to develop an identification and control design approach that can deal with this situation. Hereto, identification techniques for inferential control, uncertainty structures for robust inferential control, and appropriate control design structures are presented. As a result, a novel coordinate frame is obtained that transparently connects nominal model identification, quantification of model uncertainty, and robust inferential control, thereby enabling high performance robust inferential control.**

I. Introduction

High quality control applications introduce new control challenges such as the situation where the performance variables are not directly available for measurement during normal operating conditions. For instance, due to increasing throughput and accuracy demands, internal deformations in lightly damped electromechanical positioning systems [1] lead to a situation where dynamical behavior is present in between the measurement location and the location where performance is desired.

The explicit distinction between performance variables and measured variables motivates the need for high accuracy models, since these can be used to infer the performance variables from the measured variables. This explicit distinction, also referred to as inferential control, has important implications for modeling and control design. Firstly, for a given model, the difference between performance variables and measured variables leads to certain performance limitations as is investigated in, e.g., [2], [3], [4]. Secondly, the quality of the inferred performance variables crucially hinges on the underlying model, hence there is a need to determine models that are especially suitable for this specific control situation and a characterization of the involved model quality.

Although there are many successful applications of inferential control, especially in the area of process control [5], [6], and many important results have been obtained that enable the accurate identification of models for control, see, e.g., [7], [8] for an overview, at present it is not clear how to obtain models that are suitable for robust inferential control. Indeed, system identification techniques facilitate the accurate modeling of systems and are commonly inexpensive and fast. Initial results that address the identification of models for subsequent inferential control are presented in [9]. However, the considered identification methodology cannot deal with closed-loop operation during the identification experiment and the purpose of the model is subsequent model predictive control. In contrast, in the present paper, identification in view of subsequent robust control based on $\mathcal{H}_\infty$-optimization is considered.

System identification approaches that directly deliver a nominal model and a bound on the model uncertainty have been presented in, e.g., [10]. Although these methodologies are directly compatible with robust control based on $\mathcal{H}_\infty$-optimization, for many systems, including systems with lightly damped flexible dynamics, these methods lead to an overly large set of candidate plant models and as a consequence to an unnecessarily conservative control design.

Alternatively, the identification of the nominal model and uncertainty model can be dealt with separately. For the class of systems containing lightly damped flexible dynamics, the identification of full-order models is illusive due to the presence of parasitic phenomena. In addition, high model orders, as are required for the identification of full-order models of complex dynamics, result in computational infeasibility. This motivates the identification of control-relevant low-order models, see, e.g., [11], [12], [13], [8], that merely represent the phenomena of the true system that are relevant for subsequent control design. However, at present control-relevant identification techniques have been restricted to the case where performance variables are measured during normal operating conditions.

Addressing model uncertainty during control design is essential to guarantee that the designed controller performs well when implemented on the true system, since any real system is too complex to be represented exactly by a mathematical model, hence model inaccuracies are inevitable. In addition, when model uncertainty is addressed during controller synthesis, monotonic convergence of iterative identification and control schemes can be enforced, as is discussed in [14], [8]. Especially the use of coprime factorizations, see [15], [16], has proven successful to construct control-relevant uncertainty model structures [14], [17], [18]. However, at present these uncertainty model structures are not directly applicable to the situation where the performance variables are not measured.

Many robust control design methodologies, and as a consequence also the related control-relevant identification approaches, adopt a single-degree-of-freedom feedback configuration. However, in case the performance variables are not measured, the use of a single-degree-of-freedom feedback controller is inadequate to perform servo tasks. Although suitable controller structures, including [19, Section 5.5.1], have been developed for the inferential control situation, these controller structures cannot be used in conjunction with common $\mathcal{H}_\infty$-optimization algorithms such as [20], as is supported by the results in [21]. On the one hand, extensions to handle two-degrees-of-freedom controllers in

T. Oomen and O. Bosgra are with the Eindhoven University of Technology, Eindhoven, The Netherlands, t.a.e.oomen@tue.nl, o.h.bosgra@tue.nl. M. van de Wal is with Philips Applied Technologies, Mechatronics Department, Eindhoven, The Netherlands, m.m.j.van.de.wal@philips.com.

978-1-4244-3872-3/09/$25.00 ©2009 IEEE

robust control design methodologies have been developed in [22], [23], [24], [25], however, these extensions aim at improving tracking responses and do not address the situation where performance variables are not measured. On the other hand, $\mathcal{H}_\infty$-optimal inferential control has been considered in [26], however, the approach resorts to a specific optimization routine that is less suitable for use in conjunction with system identification.

At present, identification techniques for control, model uncertainty structures, and compatible robust control design methodologies that can deal with unmeasured performance variables are not available. The aim of the present paper is to develop, for the situation where performance variables are not measured, 1) control-relevant identification techniques, 2) model uncertainty structures, and 3) robust control design methodologies with appropriate controller structures.

The paper is organized as follows. In Section II, the inferential servo problem is defined and a motivating example is presented. In Section III, a suitable extension of the single-degree-of-freedom controller structure for robust inferential control is presented and analyzed. Then, an inferential-control-relevant identification approach that results in a specific coprime factorization of the plant is proposed in Section IV. Then, in Section V, the identified coprime factorization of the plant is employed to construct control-relevant uncertainty structures for robust inferential control. Finally, concluding remarks are presented in Section VI.

Notation: Throughout, discrete time signals and systems are considered. Generalization to continuous time systems is conceptually straightforward by appropriate, e.g., bandlimited assumptions, on signals. In addition the pair $\{N, D\}$ denotes a Right Coprime Factorization (RCF) of P if D is invertible, $N, D \in \mathcal{RH}_\infty$, $P = ND^{-1}$, and $\exists X_r, Y_r \in \mathcal{RH}_\infty$ such that the Bézout identity $X_r D + Y_r N = I$ holds. The pair $\{\bar{N}, \bar{D}\}$ is said to be a Normalized RCF (NRCF) if it is an RCF and in addition $\bar{D}^* \bar{D} + \bar{N}^* \bar{N} = I$. Dual definitions hold for Left Coprime Factorizations (LCFs) and Normalized LCFs (NLCFs), see [15]. Throughout, N and D are used exclusively to denote rational coprime factorizations over $\mathcal{RH}_\infty$. Occasionally, P is represented by the polynomial Right Matrix Fraction Description (RMFD) $P = BA^{-1}$, $B, A \in \mathbb{R}[\xi]$, i.e., polynomial matrices of appropriate sizes.

II. Problem formulation

A. Inferential servo control problem

Throughout, the linear time invariant system

$$\begin{bmatrix} z_\mathrm{p} \\ y_\mathrm{p} \end{bmatrix} = P u_\mathrm{p}, \quad P = \begin{bmatrix} P_\mathrm{z} \\ P_\mathrm{y} \end{bmatrix}, \tag{1}$$

is considered, where z_p denotes the unmeasured controlled variables, y_p are the measured variables, and u_p is the plant input. The goal of the inferential servo problem is the minimization of $z_\mathrm{ref} - z_\mathrm{p}$ given measurements y_p and reference signal z_ref by the design of an appropriate controller

$$u_\mathrm{p} = C(z_\mathrm{ref}, y_\mathrm{p}). \tag{2}$$

As an example, the inferential servo problem for a mechanical system is depicted in Figure 1. Here, the control goal is to ensure that the position of mass m_2 is close to the desired reference trajectory z_ref, given the reference signal and position measurements of mass m_1. To highlight the difference with the standard servo problem, assume that the dimensions of z_p and y_p are identical. Then, requiring mass m_1 to track z_ref, as is the case in the standard servo problem, does not imply that m_2 appropriately tracks z_ref, i.e., $z_\mathrm{ref} - y_\mathrm{p}$ small does not imply $z_\mathrm{ref} - z_\mathrm{p}$ to be small.

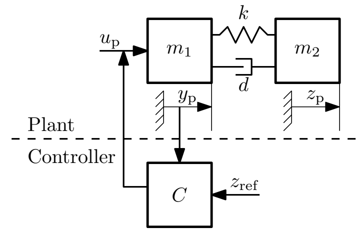

Fig. 1. Inferential servo control.

B. Identification for robust control

The following control goal is considered.

Definition 1: Given the true system P_o and interconnection defined by (1) and (2), determine

$$C^\mathrm{opt} = \arg\min_C J(P_o, C), \tag{3}$$

where $J(P, C)$ is the control criterion, which is a function of the plant P and controller C.

Since the true plant P_o is unknown, plant knowledge is reflected by a plant set $\mathcal{P}(\hat{P}, \mathbf{\Delta})$, such that $P_o \in \mathcal{P}$, where $\hat{P}$ is the nominal model and $\Delta \in \mathbf{\Delta}$ is a norm-bounded operator that represents model uncertainty. Associated with $\mathcal{P}$ is the worst-case performance cost

$$J_\mathrm{WC}(\mathcal{P}, C) = \sup_{P \in \mathcal{P}} J(P, C). \tag{4}$$

As in [27], [14], the following definition for robust-control-relevant uncertain plant set estimation is adopted.

Definition 2: Given a controller C^exp that is used during the identification experiment, determine

$$\mathcal{P}^\mathrm{RCR} = \arg\min_\mathcal{P} J_\mathrm{WC}(\mathcal{P}, C^\mathrm{exp}) \tag{5}$$

$$\text{subject to } P_o \in \mathcal{P}. \tag{6}$$

The problem in Definition 2 is dual to Definition 1, and exploits the knowledge of a stabilizing feedback controller C^exp, which is generally required to stabilize the system prior to performing the identification experiment. Since robust-control-relevance in Definition 2 depends on the particular controller, the problems in Definition 2 and Definition 1 may be solved alternately to monotonously improve the performance of the model set $\mathcal{P}(\hat{P}, \mathbf{\Delta})$, and as a consequence also the guaranteed performance for the true plant P_o.

In the present paper, first the control goal in Definition 1 is investigated in more detail. Subsequently, the identification of a plant set in Definition 2 is investigated. Firstly, a nominal model $\hat{P}$ is identified, followed by a characterization of model uncertainty Δ. By casting the model uncertainty in a specific coordinate frame, it is shown that the separate steps of nominal identification and uncertainty modeling indeed address the problem in Definition 2.

The following assumption is imposed throughout.

Assumption 3: During identification, both z_p and y_p are measured, whereas the controller only has access to y_p during normal operation of the plant.

Assumption 3 is required for system identification, since the model used to infer the performance variables should either be based on a prior knowledge or on measured data from the true plant. Assumption 3 is supported by the fact that the true system may be equipped with additional sensors during the identification experiment. It is emphasized that the resulting controller does not use measurements of z_p, since it relies only on the identified model and measurements of y_p.

III. Inferential control goal and structure selection

A. Controller structure

The inferential control problem results in certain requirements regarding the controller structure. Indeed, as is discussed in Section II, the reference signal is not defined for the measured variables. As a consequence, the single-degree-of-freedom controller structure, which generates a plant input based on an error signal, cannot be used in general.

Natural extensions of the single-degree-of-freedom controller structure include

I) indirect control [28, Section 10.4], in which case $y_\mathrm{ref} = C_1 z_\mathrm{ref}$, $u_\mathrm{p} = C_2(y_\mathrm{ref} - y_\mathrm{p})$. Basically, an intermediate reference signal is constructed for y_p that aims at minimizing $z_\mathrm{ref} - z_\mathrm{p}$.

II) inferential control [5], in which case $\hat{z}_\mathrm{p} = C_1(u_\mathrm{p}, y_\mathrm{p})$, $u_\mathrm{p} = C_2(z_\mathrm{ref} - \hat{z}_\mathrm{p})$. In this case, the feedback controller is based on an estimate $\hat{z}_\mathrm{p}$ of z_p.

Although these indirect and inferential controller structures can deal with the inferential servo control problem, these approaches lead to structured control problems in the standard plant configuration. As a consequence, the resulting controller synthesis problem cannot be dealt with using standard optimal controller synthesis algorithms, which is also observed in the design of a two-degree-of-freedom feedback controller in [25]. In this case, it is not straightforward to connect the identification and control criteria as is required in case of identification for inferential control.

To enable the use of standard controller synthesis algorithms to solve the inferential servo problem, a general two-degree-of-freedom controller

$$u_\mathrm{p} = \underbrace{\begin{bmatrix} C_1 & C_2 \end{bmatrix}}_{=:C} \begin{bmatrix} z_\mathrm{ref} \\ y_\mathrm{p} \end{bmatrix} \tag{7}$$

is employed. In addition to a combined feedforward/feedback configuration, the controller structure (7) includes the indirect and inferential controller structures as special cases.

It is emphasized that the structures indicated by I and II, above, are not necessarily identical. Indeed, for the indirect situation I, C_1 must be stable, otherwise y_ref may become unbounded. In contrast, using the parameterization (7), C must be stabilized by P, hence there is no open-loop stability requirement on C_1.

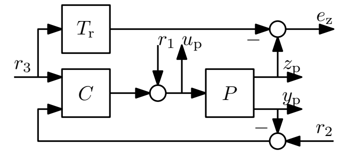

Fig. 2. Unweighted exogenous inputs and outputs.

B. Selection of exogenous inputs and outputs

The definition of the control criterion $J(P, C)$ in Definition 1 requires a selection of exogenous inputs and outputs. These exogenous inputs and outputs have to be carefully introduced to ensure that the resulting controller is useful. For instance, in view of the inferential control application in Section II, the controller should enforce tracking of a reference signal in the presence of disturbances and measurement noise. To enable a general weighting filter selection, the (unweighted) exogenous inputs and outputs are selected as depicted in Figure 2. Here, r_1 represents disturbances at the plant input that have to be attenuated, r_2 represents measurement noise, and r_3 represents the reference signal. In addition, e_z is the tracking error, y_p is the plant output, which is included to enforce internal stability, and u_p is the plant input, which is included to penalize a large control effort.

The choice of exogenous signals in Figure 2 results in the mapping $\overline{M} : \overline{w} \mapsto \overline{z}$, where

$$\overline{M} = \begin{bmatrix} P_\mathrm{z} \\ P_\mathrm{y} \\ I \end{bmatrix} (I + C_2 P_\mathrm{y})^{-1} \begin{bmatrix} C_1 & C_2 & I \end{bmatrix} - \begin{bmatrix} \overline{T}_\mathrm{r} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

$$\overline{w} = \begin{bmatrix} r_3 \\ r_2 \\ r_1 \end{bmatrix}, \quad \overline{z} = \begin{bmatrix} e_\mathrm{z} \\ y_\mathrm{p} \\ u_\mathrm{p} \end{bmatrix}. \tag{9}$$

In (8), $\overline{T}_\mathrm{r} \in \mathcal{RH}_\infty$ is a reference model, which is generally required to ensure a sensible problem formulation.

To formulate internal stability requirements, let

$$\begin{bmatrix} P_\mathrm{z} \\ P_\mathrm{y} \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C_\mathrm{z} & D_\mathrm{z} \\ C_\mathrm{y} & D_\mathrm{y} \end{array}\right], \ \begin{bmatrix} C_1 & C_2 \end{bmatrix} = \left[\begin{array}{c|cc} A_\mathrm{c} & B_{\mathrm{c},1} & B_{\mathrm{c},2} \\ \hline C_\mathrm{c} & D_{\mathrm{c},1} & D_{\mathrm{c},2} \end{array}\right] \tag{10}$$

be minimal realizations of P and C, respectively, and assume that the feedback loop of P and C is well-posed. Internal stability is now characterized as follows.

Proposition 4: The system given by the interconnection (1) and (7) is internally stable, i.e., for $z_\mathrm{ref} = 0$ the states corresponding to P and C in (10) tend to zero for all initial states, if and only if $\overline{M} \in \mathcal{RH}_\infty$.

Proof: A proof follows along similar lines as in [29, Lemma 5.3] and employing the fact that $\overline{T}_\mathrm{r} \in \mathcal{RH}_\infty$. ∎

C. Standard plant formulation

To systematically formulate the inferential control problem such that it can be dealt with using standard algorithms, the standard plant formulation is employed. By extracting the controller from the setup in Figure 2, the (unweighted) standard plant

$$\begin{bmatrix} \overline{z} \\ y \end{bmatrix} = \bar{G}(P) \begin{bmatrix} \overline{w} \\ u \end{bmatrix}, \tag{11}$$

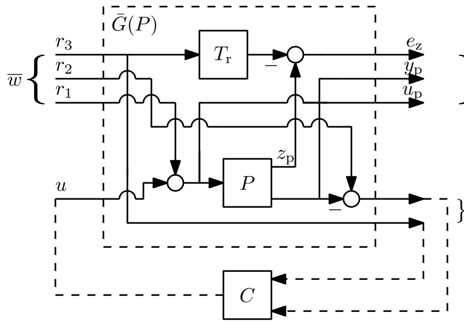

Fig. 3. Recasting the setup of Figure 2 into the standard plant configuration.

is obtained, see Figure 3. In addition, the weighting matrices

$$W = \begin{bmatrix} W_{\rm e} & 0 & 0 \\ 0 & W_{\rm y} & 0 \\ 0 & 0 & W_u \end{bmatrix}, \quad V = \begin{bmatrix} V_3 & 0 & 0 \\ 0 & V_2 & 0 \\ 0 & 0 & V_1 \end{bmatrix}, \quad (12)$$

where $W, W^{-1}, V, V^{-1} \in \mathcal{RH}_\infty$ such that

$$z = W\overline{z}, \quad \overline{w} = Vw, \quad (13)$$

are introduced to specify the performance specifications. Next, the operator

$$M(P,C) : w \mapsto z = W\mathcal{F}_l(G(P), C)V \quad (14)$$

is introduced, leading to the control criterion, see Definition 1,

$$J(P,C) = \|M(P,C)\|_\infty . \quad (15)$$

Clearly, employing an $\mathcal{H}_\infty$-norm in (3) enforces C to be internally stabilizing. Additionally, it enables the incorporation of model uncertainty in the design procedure, as is discussed in Section V. The specific choice of controller structure in (7) enables the use of standard controller synthesis algorithms to perform the minimization in (3).

IV. INFERENTIAL-CONTROL-RELEVANT IDENTIFICATION

A. Identification criterion

Since the true plant P_o is unknown, the optimization in (3) cannot be performed directly. This knowledge is reflected by a model $\hat{P}$. The performance of a controller C that is optimal for $\hat{P}$ can degrade when this controller is implemented on P_o. In the case of inferential control, the performance degradation can be caused by model errors both in the dynamical behavior in the feedback path determined by $P_{\rm y}$ and in the dynamical behavior corresponding to the performance variables that is determined by $P_{\rm z}$. This performance degradation is upper bounded in the following proposition.

Proposition 5: The criterion $\|M(P_o, C)\|_\infty$ is upper bounded by

$$\|M(P_o,C)\|_\infty \leq \|M(\hat{P},C)\|_\infty + \|M(P_o,C) - M(\hat{P},C)\|_\infty \quad (16)$$

Proof: Follows directly by application of the triangle inequality, see also [11]. ■

The first term on the right-hand-side of (16) represents a nominal control design, whereas the second term represents performance degradation. For a given internally stabilizing controller $C^{\rm exp}$, this term represents a control-relevant identification criterion.

Definition 6: The control-relevant identification criterion is defined as

$$\|M(P_o, C^{\rm exp}) - M(\hat{P}, C^{\rm exp})\|_\infty, \quad (17)$$

which should be minimized over $\hat{P}$.

In (17), $M(P_o, C^{\rm exp})$ depends on the true system and can be identified from data[1]. In contrast to the case where all controlled variables (z) are also measured variables (y), the inferential case inevitably requires a (temporary) measurement of the inferential variables during the identification step.

It is emphasized that, in contrast to common iterative identification and control design techniques, Definition 6 involves a nine-block problem to guarantee internal stability using the two-degrees-of-freedom controller (7).

B. Recasting as coprime factorization

In this section, the control-relevant identification criterion (17) is recast as an optimization over coprime factorizations. These coprime factorizations are essential for the construction of control-relevant uncertainty models in Section V. Additionally, it significantly reduces the complexity of the optimization problem.

The following result is required throughout.

Theorem 7 (LCF with co-inner numerator): Let $H(z) \in \mathcal{R}^{n_z \times n_w}$, $n_w \geq n_z$ with minimal state-space realization (A, B, C, D), assuming D has full row rank. Then, there exists an LCF of H, $H = \tilde{D}^{-1}\tilde{N}$ such that $\tilde{N}$ is co-inner, i.e., $\tilde{N}\tilde{N}^* = I$, if and only if $H(e^{j\omega})H^{\sim}(e^{j\omega}) > 0 \; \forall \omega \in [0, 2\pi]$.

See [30] for a proof of Theorem 7 and state-space formulas for the computation of an LCF with co-inner numerator.

The following proposition is the main result of this section.

Proposition 8: Consider the control-relevant identification criterion (17), where W and V are defined in (12), and let $\{\tilde{N}_e, \tilde{D}_e\}$ be an LCF with co-inner numerator of $\begin{bmatrix} C_1V_3 & C_2V_2 & V_1 \end{bmatrix}$, where $\tilde{N}_e = \begin{bmatrix} \tilde{N}_{e,3} & \tilde{N}_{e,2} & \tilde{N}_{e,1} \end{bmatrix}$. Then, the $\mathcal{H}_\infty$-norm in (17) is equal to

$$\left\| W \left(\begin{bmatrix} N_{z,o} \\ N_o \\ D_o \end{bmatrix} - \begin{bmatrix} \hat{N}_z \\ \hat{N} \\ \hat{D} \end{bmatrix} \right) \right\|_\infty, \quad (18)$$

where

$$\begin{bmatrix} N_z \\ N \\ D \end{bmatrix} = \begin{bmatrix} P_{\rm z} \\ P_{\rm y} \\ I \end{bmatrix} (\tilde{D}_e + \tilde{N}_{e,2}V_2^{-1}P_{\rm y})^{-1}. \quad (19)$$

Proof: Firstly, observe that by substituting (8) in (17) yields

$$\left\| W \left(\begin{bmatrix} P_{{\rm z},o} \\ P_{{\rm y},o} \\ I \end{bmatrix} (I + C_2P_{{\rm y},o})^{-1} - \begin{bmatrix} \hat{P}_{\rm z} \\ \hat{P}_{\rm y} \\ I \end{bmatrix} (I + C_2\hat{P}_{\rm y})^{-1} \right) \tilde{D}_e^{-1}\tilde{N}_e \right\|_\infty, \quad (20)$$

where the reference model $T_{\rm r}$ cancels out. In virtue of co-innerness of $\tilde{N}_e$, $\tilde{N}_e$ is norm-preserving, hence (20) directly yields the desired result (18). ■

The following intermediate result is required to prove that (19) contains an RCF of $P_{\rm y}$.

[1]In the sequel, the superscript exp is often omitted to facilitate the notation.

Proposition 9: Let $\{\tilde{N}_e, \tilde{D}_e\}$ be an LCF of $\begin{bmatrix} C_1V_3 & C_2V_2 & V_1 \end{bmatrix}$, where $\tilde{N}_e = \begin{bmatrix} \tilde{N}_{e,3} & \tilde{N}_{e,2} & \tilde{N}_{e,1} \end{bmatrix}$ and assume that $\begin{bmatrix} C_2V_2 & V_1 \end{bmatrix}$ has full row rank. Then, $\tilde{N}_{e,2}$ and $\tilde{N}_{e,1}$ are left coprime if and only if $\begin{bmatrix} C_2V_2 & V_1 \end{bmatrix}$ has no transmission zeros outside the unit disc.

Since V_1^{-1} is assumed to exist, $\begin{bmatrix} CV_2 & V_1 \end{bmatrix}$ has full row rank. In addition, since $V_1^{-1} \in \mathcal{RH}_\infty$ by assumption, $\tilde{N}_{e,2}$ and $\tilde{N}_{e,1}$ are left coprime in virtue of Proposition 9.

Proposition 10: Let $M(P,C) \in \mathcal{RH}_\infty$ and let $\{\tilde{N}_e, \tilde{D}_e\}$ be an LCF of $\begin{bmatrix} C_1V_3 & C_2V_2 & V_1 \end{bmatrix}$. Then, $\{N, D\}$ in (19) is an RCF of P_{y} and in addition $N_z \in \mathcal{RH}_\infty$.

The specific coprime factorization of P in Proposition 8 is unique up to right multiplication by a constant unitary matrix. Control-relevance of the specific coprime factorization is twofold. Firstly, the specific coprime factor realization uniquely reduces the nine-block problem control-relevant criterion in (17) to a three-block problem (18) under preservation of norms. Secondly, as will be shown in Section V, the specific coprime factor realization leads to a coordinate frame for representing model uncertainty that is robust-control-relevant in the sense of Definition 2.

C. Frequency domain identification algorithm

1) Frequency domain identification: The identification of coprime factors in (18) cannot be solved directly from measured data since calculation of the $\mathcal{H}_\infty$-norm requires knowledge of the true system. The frequency domain interpretation of the $\mathcal{H}_\infty$-norm is exploited to formulate a tractable identification problem. Since identification in practice is always performed over a finite time interval, a discrete frequency grid Ω is used in frequency domain identification.

Proposition 11: Consider the discrete frequency grid Ω. Then, the identification criterion (17) is lower bounded by

$$\max_{\omega_i \in \Omega} \bar{\sigma} \left(W \left(\begin{bmatrix} N_{z,o} \\ N_o \\ D_o \end{bmatrix} - \begin{bmatrix} \hat{N}_z \\ \hat{N} \\ \hat{D} \end{bmatrix} \right) \right) \tag{21}$$

subject to $M(\hat{P}, C) \in \mathcal{RH}_\infty$

Proof: Follows by considering the $\mathcal{H}_\infty$-norm. ■

The identification criterion in (21) ensures that the resulting optimization problem is tractable, since under Assumption 3, $\begin{bmatrix} N_{z,o}^T & N_o^T & D_o^T \end{bmatrix}^T$ can be identified from data by employing frequency domain identification techniques.

2) Coprime factor parameterization: A suitable parameterization of $\hat{N}_z$, $\hat{N}$, $\hat{D}$ is required to 1) satisfy the stability constraint in (21); 2) ensure stable and low-order factors $\hat{N}_z$, $\hat{N}$, and $\hat{D}$; 3) enable the use of efficient optimization algorithms. In this section, an extension of the algorithm in [18] towards inferential control is proposed, where P is parameterized as a canonical right matrix fraction description that is linear in the parameters θ, i.e.,

$$P = \begin{bmatrix} C(\theta) \\ B(\theta) \end{bmatrix} A(\theta)^{-1}, \tag{22}$$

which in conjunction with (19) yields the expression

$$\begin{bmatrix} \hat{N}_z \\ \hat{N} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} C(\theta) \\ B(\theta) \\ A(\theta) \end{bmatrix} (\tilde{D}_e A(\theta) + \tilde{N}_{e,2} V_2^{-1} B(\theta))^{-1}. \tag{23}$$

In case P is internally stabilized by C, then expression (23) contains a parametrization of coprime factors, see [18].

The minimization over θ in (21) can be achieved by employing Lawson's algorithm in conjunction with a generalization of the Sanathanan-Koerner and Gauss-Newton iterations as is described in [18]. In addition, a data-based inner product is employed in [18] to ensure numerically reliable computations.

The parameterization (23) exploits prior knowledge of the stabilizing controller C^{exp} and weighting filter V. As a consequence, the resulting coprime factors enable the construction of a model uncertainty coordinate frame that is robust-control-relevant in the sense of Definition 2. Clearly, this may increase the McMillan degree of the coprime factors compared to the plant P, and thus also compared to normalized coprime factorizations. However, the additional complexity is essential to construct robust-control-relevant model sets.

V. Uncertainty structures for robust inferential control

The identified model in Section IV is not an exact representation of reality. Especially in the case of inferential control, where the controlled variables z are only measured during the identification experiment and inaccessible during normal operation of the plant, model quality is a crucial aspect during control design. In this section, control-relevant uncertainty model structures are presented that are suitable for inferential servo control.

In view of robust control based on $\mathcal{H}_\infty$-optimization, a necessary requirement regarding perturbation models is that the true system should be in the model set for a certain norm-bounded perturbation. Additionally, a desirable property of the model set is that it enables a high performance control design, i.e., the model set should not unnecessarily include candidate plants that are difficult to control. The observation that both P_o and $\hat{P}$ are stabilized by C during the identification experiment leads to the following result, which is the main result of this section.

Proposition 12: Let C internally stabilize P, and consider $N_z, N, D \in \mathcal{RH}_\infty$ in (19), where $\hat{N}, \hat{D}$ are coprime, and let $\{N_{c,2}, D_{c,22}\}$ be an RCF of C_2. Then, all plants P that are stabilized by C are given by

$$P = \begin{bmatrix} N_z + W_{\mathrm{e}}^{-1}\Delta_z \\ N + D_{c,22}\Delta_u \end{bmatrix} (D - N_{c,2}\Delta_u)^{-1}, \tag{24}$$

where

$$\Delta = \begin{bmatrix} \Delta_z \\ \Delta_u \end{bmatrix} \in \mathcal{RH}_\infty \tag{25}$$

The novelty of Proposition 12 is that it generalizes the well-known dual-Youla-Kučera parameterization, see, e.g., [17], to plants with unmeasured performance variables and two-degrees-of-freedom controllers.

The purpose of W_{e}^{-1} in (24) is to enforce control relevance of the uncertainty structure, see Proposition 14. In addition, to show that the factorization in Proposition 8 leads to a control-relevant structure, the following coprime factorization of C_2 is required.

Definition 13: The pair $\{N_{c,2}, D_{c,22}\}$ is an (W_u, W_y)-normalized RCF of C_2 if it is an RCF of C_2 and in addition,

$$\begin{bmatrix} W_u N_{c,2} \\ W_y D_{c,22} \end{bmatrix}^* \begin{bmatrix} W_u N_{c,2} \\ W_y D_{c,22} \end{bmatrix} = I. \tag{26}$$

State-space formulas for computing the factorizations in (26) are provided in [27].

Next, suppose that a certain norm-bound γ is selected, e.g., as the result of a model validation procedure [32]. Then, the set of candidate plants is given by

$$\mathcal{P}_\gamma = \left\{ P \middle| P = \begin{bmatrix} \hat{N}_z + W_e^{-1}\Delta_z \\ \hat{N} + D_{c,22}\Delta_u \end{bmatrix} (\hat{D} - N_{c,2}\Delta_u)^{-1}, \left\| \begin{bmatrix} \Delta_z \\ \Delta_u \end{bmatrix} \right\| < \gamma \right\}. \tag{27}$$

Associated with $\mathcal{P}_\gamma$ is the worst-case performance measure

$$J_{\text{WC}}(\mathcal{P}_\gamma, C) = \sup_{P \in \mathcal{P}_\gamma} J(P, C) \tag{28}$$

The following proposition reveals control-relevance of the previously presented coprime factorization.

Proposition 14: Consider the perturbation model structure (27), where $\hat{N}_z, \hat{N}, \hat{D}, N_{c,2}, D_{c,22}$ are as proposed in Proposition 8 and Definition 13. Then,

$$J_{\text{WC}} \leq J(\hat{P}, C) + \left\| \begin{bmatrix} \Delta_z \\ \Delta_u \end{bmatrix} \right\|_\infty \tag{29}$$

The coordinate frame that is obtained by the specific factorizations in Proposition 8 and Definition 13 result in a transparent connection between nominal model identification, model uncertainty quantification, and robust control design. Specifically, the norm of $\bar{\Delta}$ directly affects the worst-case performance in (29), resulting in a robust-control-relevant model set in the sense of Definition 2. In contrast, other coprime factorizations generally lead to an overly large candidate plant set and as a result also to an unnecessarily conservative control design. To quantify the remaining model uncertainty such that (6) holds, any model uncertainty quantification procedure may be applied.

VI. Conclusions

Control-relevant identification and uncertainty model structures based on coprime factorizations have been presented for robust inferential control. The presented results enable the design of robust controllers in the case that not all the performance variables (z) are included in the set of measured variables (y). The presented framework thus extends previous results in identification for control and uncertainty model structures for robust control to the inferential control case. In addition, in the paper, controller structures have been presented for inferential control design approaches based on $\mathcal{H}_\infty$-optimization.

References

[1] M. van de Wal, G. van Baars, F. Sperling, and O. Bosgra, "Multivariable $\mathcal{H}_\infty/\mu$ feedback control design for high-precision wafer stage motion," *Contr. Eng. Prac.*, vol. 10, no. 7, pp. 739–755, 2002.

[2] J. Hong and D. S. Bernstein, "Bode integral constraints, colocation, and spillover in active noise and vibration control," *IEEE Trans. Contr. Syst. Techn.*, vol. 6, no. 1, pp. 111–120, 1998.

[3] J. S. Freudenberg, C. V. Hollot, R. H. Middleton, and V. Toochinda, "Fundamental design limitations of the general control configuration," *IEEE Trans. Automat. Contr.*, vol. 48, no. 8, pp. 1355–1370, 2003.

[4] V. Kariwala and S. Skogestad, "Limits of disturbance rejection for indirect control," in *Proc. 44th Conf. Dec. Contr.*, Seville, Spain, 2005, pp. 3620–3625.

[5] J. R. Parrish and C. B. Brosilow, "Inferential control applications," *Automatica*, vol. 21, no. 5, pp. 527–538, 1985.

[6] F. J. Doyle, III, "Nonlinear inferential control for process applications," *J. Proc. Cont.*, vol. 8, no. 5-6, pp. 339–353, 1998.

[7] P. M. J. Van den Hof and R. J. P. Schrama, "Identification and control - closed-loop issues," *Automatica*, vol. 31, no. 12, pp. 1751–1770, 1995.

[8] H. Hjalmarsson, "From experiment design to closed-loop control," *Automatica*, vol. 41, pp. 393–438, 2005.

[9] R. Amirthalingam and J. H. Lee, "Subspace identification based inferential control applied to a continuous pulp digester," *J. Proc. Cont.*, vol. 9, no. 5, pp. 397–406, 1999.

[10] J. Chen and G. Gu, *Control-Oriented System Identification: An $\mathcal{H}_\infty$ Approach*. New York, NY, USA: John Wiley & Sons, 2000.

[11] R. J. P. Schrama, "Accurate identification for control: The necessity of an iterative scheme," *IEEE Trans. Automat. Contr.*, vol. 37, no. 7, pp. 991–994, 1992.

[12] M. Gevers, "Towards a joint design of identification and control ?" in *Essays on Control : Perspectives in the Theory and its Applications*, H. L. Trentelman and J. C. Willems, Eds. Boston, MA, USA: Birkhäuser, 1993, ch. 5, pp. 111–151.

[13] P. Albertos and A. Sala, Eds., *Iterative Identification and Control*. London, UK: Springer, 2002.

[14] R. A. de Callafon and P. M. J. Van den Hof, "Suboptimal feedback control by a scheme of iterative identification and control design," *Math. Mod. Syst.*, vol. 3, no. 1, pp. 77–101, 1997.

[15] M. Vidyasagar, *Control System Synthesis: A Factorization Approach*. Cambridge, MA, USA: The MIT Press, 1985.

[16] D. C. McFarlane and K. Glover, *Robust Controller Design Using Normalized Coprime Factor Plant Descriptions*, ser. LNCIS. Berlin, Germany: Springer-Verlag, 1990, vol. 138.

[17] S. G. Douma and P. M. J. Van den Hof, "Relations between uncertainty structures in identification for robust control," *Automatica*, vol. 41, pp. 439–457, 2005.

[18] T. Oomen and O. Bosgra, "Robust-control-relevant coprime factor identification: A numerically reliable frequency domain approach," in *Proc. 2008 Americ. Contr. Conf.*, Seattle, WA, USA, 2008, pp. 625–631.

[19] P. Albertos and A. Sala, *Multivariable Control Systems: An Engineering Approach*. London, UK: Springer-Verlag, 2004.

[20] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State-space solutions to standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems," *IEEE Trans. Automat. Contr.*, vol. 34, no. 8, pp. 831–947, 1989.

[21] J. M. Edmunds, "Output response tuning of one- and two-degree-of-freedom controllers using an $\mathcal{H}_\infty$ approach," *Int. J. Contr.*, vol. 59, no. 4, pp. 1119–1126, 1994.

[22] D. J. Hoyle, R. A. Hyde, and D. J. N. Limebeer, "An $\mathcal{H}_\infty$ approach to two degree of freedom design," in *Proc. 30th Conf. Dec. Contr.*, Brighton, UK, 1991, pp. 1581–1585.

[23] D. J. N. Limebeer, E. M. Kasenally, and J. D. Perkins, "On the design of robust two degree of freedom controllers," *Automatica*, vol. 29, no. 1, pp. 157–168, 1993.

[24] A. Dehghani, A. Lanzon, and B. D. O. Anderson, "A two-degree-of-freedom $\mathcal{H}_\infty$ control design method for robust model matching," *Int. J. Rob. Nonlin. Contr.*, vol. 16, no. 10, pp. 467–483, 2006.

[25] I. Yaesh and U. Shaked, "Two-degree-of-freedom $\mathcal{H}_\infty$-optimization of multivariable feedback systems," *IEEE Trans. Automat. Contr.*, vol. 36, no. 11, pp. 1272–1276, 1991.

[26] M. J. Grimble, "$3\frac{1}{2}$ DOF polynomial solution of the inferential $\mathcal{H}_2/\mathcal{H}_\infty$ control problem with application to metal rolling," *J. Dyn. Syst., Meas., and Contr.*, vol. 120, no. 4, pp. 445–455, 1998.

[27] T. Oomen, R. van Herpen, and O. Bosgra, "Robust-control-relevant coprime factor identification with application to model validation of a wafer stage," in *15th IFAC Symp. Sys. Id.*, Saint-Malo, France, 2009, pp. 1044–1049.

[28] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*, 2nd ed. West Sussex, UK: John Wiley & Sons, 2005.

[29] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Upper Saddle River, NJ, USA: Prentice Hall, 1996.

[30] C.-C. Chu, "On discrete inner-outer and spectral factorizations," in *Proc. 7th Americ. Contr. Conf.*, Atlanta, GA, USA, 1988, pp. 1699–1700.

[31] K. Zhou and Z. Ren, "A new controller architecture for high performance, robust, and fault-tolerant control," *IEEE Trans. Automat. Contr.*, vol. 46, no. 10, pp. 1613–1618, 2001.

[32] T. Oomen and O. Bosgra, "Estimating disturbances and model uncertainty in model validation for robust control," in *Proc. 47th Conf. Dec. Contr.*, Cancún, Mexico, 2008, pp. 5513–5518.