Nonlinear feedforward control for a class of tasks: A Gaussian Process approach applied to a printer

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1 Background

Feedforward control is essential for accurate reference tracking in motion control. For modern systems, feedforward controllers involve the inverse of increasingly complex systems with high order dynamics and dominant nonlinear dynamics of unknown structure. In addition, a large class of tasks must be performed.

2 Problem formulation

The aim is to obtain a nonlinear, noncausal feedforward controller for systems with complex nonlinear dynamics, which is applicable to a range of references. To this end, the system is parametrized as a GP. This allows for high flexibility and the specification of relevant prior information. Gaussian Process (GP) regression has had a major impact on the field of system identification [2] and control [1], as its nonparametric, possibly nonlinear model structure allows for the representation of a large range of systems.

3 Kernel-based inverse model control

Let $G(q)$ denote a discrete-time, nonlinear SISO system, such that G^{-1} characterizes the control effort $u(t)$ solely from past and future outputs $y(t+\tau)$, $t,\tau \in \mathbb{Z}$. In particular, G^{-1} is assumed to be a non-causal nonlinear impulse response (NFIR) system of the form

$$u(t) = f(\mathbf{y}_t), \tag{1}$$

with $\mathbf{y}_t = [y(t+n_{ac}), \ldots, y(t-n_c)]^\top$. Given a stabilizing feedback controller $C(q)$, the aim is to model f from a dataset $\mathcal{D} = \{u(t), y(t)\}$ such that $f(\mathbf{r}_t)$ yields the control effort $u(t)$ that realizes an arbitrary reference sequence $\mathbf{r}_t = [r(t+n_{ac}), \ldots, r(t-n_c)]^\top$.
The key idea is to model f as a GP, i.e.,

$$f(\mathbf{y}_t) \sim \mathcal{GP}(0, k(\mathbf{y}_{t_1} - \mathbf{y}_{t_2})), \tag{2}$$

where k is a kernel function that poses a prior on the smoothness of f with respect to $\mathbf{y}$, e.g., a Matèrn$_{3/2}$ kernel [3]. The expected feedforward signal required to realize reference $R = [\mathbf{r}_1, \ldots, \mathbf{r}_N]^\top$ is then given by the posterior mean of the GP as

$$\mathbb{E}[\mathbf{f}(R)] = K(R,Y)\left[K(Y,Y) + \sigma_n^2 I\right]^{-1}\mathbf{u}. \tag{3}$$

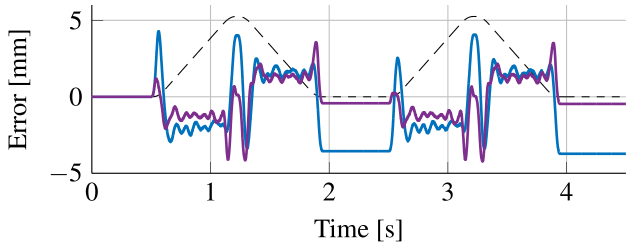

Figure 1: Error $e = r - y$ using linear feedforward (—) and the GP-based feedforward signal (—), along with the scaled reference (- -). With GP-based feedforward, $\|e\|_2$ is reduced by a factor 1.9.

This allows for the synthesis of feedforward signals for different tasks r, given that $\mathcal{D}$ contains observations of similar trajectories y.

4 Experimental results

The developed feedforward approach is applied to an A3 printer with friction. First, 11 closed-loop experiments are performed to obtain a dataset, using standard acceleration and velocity feedforward. In each experiment, a different reference $\tilde{r}_j = a_j r_1$ is used, with r_1 a second order reference and $a_j \in [0.90, 0.92, \ldots, 1.10]$. The data-set is then used to construct a feedforward signal with (3), for a different reference r_2. The result is shown in Figure 1. Even though r_2 is not used as a reference when obtaining the dataset, the error is reduced substantially by the GP-based approach.

5 Conclusion

A feedforward approach is presented that generates feedforward signals for systems with nonlinear dynamics of unknown structure while allowing for task flexibility. Future work aims at the extension towards MIMO systems.

References

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