

## Data-Driven Compensation of Unmodeled Dynamics for

### Complex Mechatronic Systems – Part II: Combining Models and Neural Networks

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#### Abstract

Unknown, typically nonlinear dynamics in complex mechatronic systems often limit the tracking performance of feedforward control based on physical models. This research focuses on compensating these unknown dynamics through complementing the physical model with a neural network in such a way that the neural network only compensates dynamics that cannot be captured by the model. This results not only in increased performance, but additionally in interpretable model parameters, neural networks with fewer parameters, and small neural network outputs. The approach is validated on the Allura Centron, an interventional X-ray developed by Philips, for which the tracking performance is improved by a factor two and five for two respective axes.

#### Theory [1]

The feedforward signal  $f$  as a function of the reference  $r$  consists of the parallel combination of a physical model  $\mathcal{M}_\theta$  with parameters  $\theta$ , and a neural network  $\mathcal{C}_\varphi$ , i.e.,

$$f = \mathcal{M}_\theta(r) + \mathcal{C}_\varphi(r).$$

The model encapsulates all that is known about the system dynamics. It can range from simple mass-damping feedforward, i.e.,  $\mathcal{M}_{\theta=(m,b)}(r) = m\ddot{r} + b\dot{r}$ , to multibody models capturing complex kinematics. The neural network is a universal approximator and in this work is given by

$$\mathcal{C}_\varphi(r) = W_L \sigma(W_{L-1} \sigma(\dots \sigma(W_0 r + b_0) \dots)) + b_{L-1}, \quad \varphi = \{W_L, W_{L-1}, \dots, W_0, b_{L-1}, \dots, b_0\},$$

with  $\sigma(\cdot)$  an activation function such as  $\tanh(x)$ ,  $W_i$  weight matrices, and  $b_i$  bias vectors.

The neural network can also approximate the dynamics included in the model (e.g., it is very easy to approximate a straight line  $m\ddot{r}$ ). Consequently, the contributions of  $\mathcal{M}_\theta(r)$  and  $\mathcal{C}_\varphi(r)$  can be interchanged, such that modelled dynamics can end up in  $\mathcal{C}_\varphi(r)$ , resulting in uninterpretable model coefficients and big neural network contributions. Thus, the contribution  $\mathcal{C}_\varphi(r)$  to  $f$  that could have

been explained by  $\mathcal{M}_\theta(r)$  is regularized using orthogonal projections. It is assumed that the model parameters  $\theta$  appear linearly in the model, such that the model is a weighted sum of basis functions of  $r$ , i.e.,

$$\mathcal{M}_\theta(r) = M(r)\theta.$$

Then, interpreting  $r$  as a vectorized signal in  $R^N$ , such that  $M(r) \in R^{N \times N_\theta}$ , the contribution of  $\mathcal{C}_\varphi(r)$  to dynamics that could be captured by  $\mathcal{M}_\theta(r)$ , i.e., for any  $\theta$ , is given by

$$\mathcal{C}_\varphi^M(r) = M(r)M^+(r)\mathcal{C}_\varphi(r) = U_1U_1^T(r)\mathcal{C}_\varphi(r),$$

in which the last equality holds by singular value decomposition  $M(r) = U_1(r)\Sigma(r)V_1^T(r)$ . The term  $U_1U_1^T(r)\mathcal{C}_\varphi(r)$  can be recognized as the orthogonal projection of  $\mathcal{C}_\varphi(r)$  onto the image of matrix  $M(r)$ , represented by basis  $U_1(r)$ . During training, it is then ensured that the neural network does not generate contributions in the subspace spanned by the model, i.e., the parallel parametrization is optimized according to

$$\theta^*, \varphi^* = \arg \min_{\theta, \varphi} \left\| f - \left( \mathcal{M}_\theta(r) + \mathcal{C}_\varphi(r) \right) \right\|_2^2 + \lambda \left\| U_1U_1^T(r)\mathcal{C}_\varphi(r) \right\|_2^2,$$

in which the regularization ensures that contributions of  $\mathcal{C}_\varphi$  which could have been explained by  $\mathcal{M}_\theta$ , are penalized, resulting in interpretable model parameters  $\theta$  and smaller neural network forces.

### Application to medical X-ray

The approach is applied to the Allura Centron, an interventional X-ray developed by Philips, see figure 1. It consists of three axes of rotation, labelled  $\theta_1, \theta_2, \theta_3$ , that together allow for positioning the source and detector of the X-ray to enable flexible and accurate imaging. The nonlinear kinematics can be accurately captured by models. However, hard-to-model nonlinearities limit the effectiveness of model-based feedforward control and tracking performance. For example, the cable connecting to the  $\theta_3$ -axis causes a complex position-varying stiffness and inertia. Even worse, the cable connecting to the  $\theta_2$ -axis can get stuck between the base frame and the C-arc, resulting in a position- and direction-varying nonlinear resistance to motion. Lastly, the  $\theta_3$ -axis has complex position-varying friction characteristics due to variations of the contact force with the rollers guiding its movement.

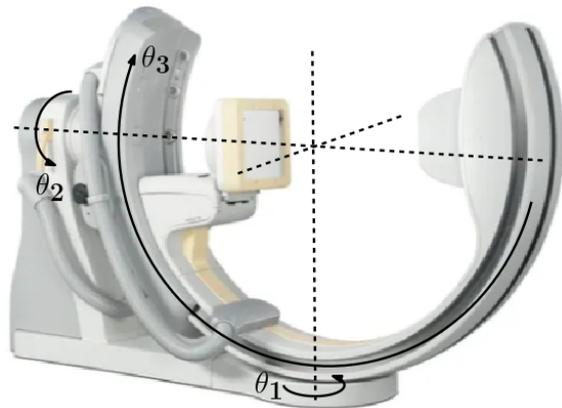


Figure 1: Allura Centron interventional X-ray

Table 1: Performance norms for  $\theta_3$ -axis

Norm	Feedback	Model	Parallel
MAE/deg	0.402	0.095	0.020
RMSE/deg	0.521	0.116	0.029
Inf/deg	1.400	0.290	0.027

Table 2: Performance norms for  $\theta_2$ -axis

Norm	Feedback	Model	Parallel
MAE/deg	0.428	0.072	0.031
RMSE/deg	0.507	0.089	0.046
Inf/deg	1.072	0.351	0.423

A dataset is obtained using feedback forces and corresponding outputs, and both a purely model-based parametrization and parallel parametrization consisting of a model and neural network are optimized based on this dataset. The parallel parametrization is optimized according to the orthogonal projection-based cost function. These optimized parametrizations are evaluated for a new reference. The resulting error norms for both the  $\theta_2$ - and  $\theta_3$ -axis are shown in table 1 and 2 respectively. These tables convey that the proposed approach is able to reduce the mean-average error (MAE) and root-mean-square error (RMS) by a factor two in the  $\theta_2$ -axis, and by a factor five in the  $\theta_3$ -axis. The observed increase in infinity norm for the  $\theta_2$ -axis is due to transients.

Additionally, the input generated by the model and parallel parametrization are compared to the desired input for the  $\theta_3$ -axis in figure 2. It is observed that the parallel parametrization with the neural network is able to capture the desired input, whereas the model is not flexible enough to capture the position-dependent damping characteristics. The resulting error at a (different) representative interval is visualized in figure 3, which shows that the error is significantly reduced by the parallel parametrization compared to just model-based feedforward.

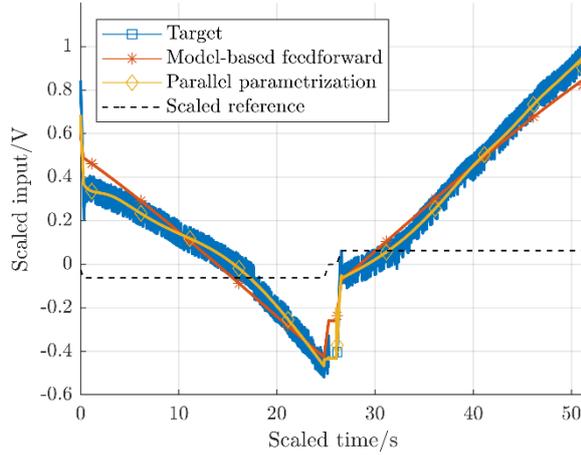


Figure 2: Generated and desired input for the  $\theta_3$ -axis by model and parallel parametrization.

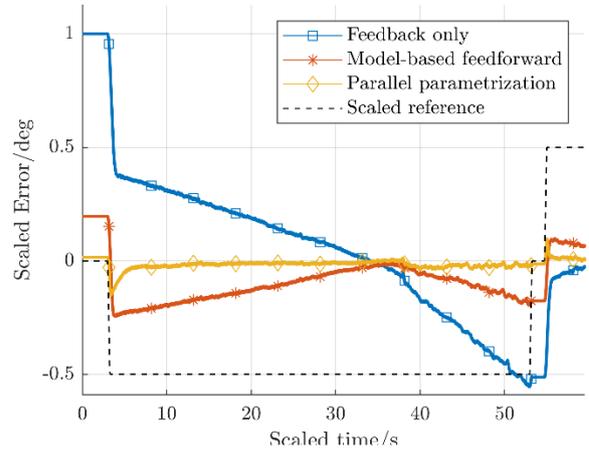


Figure 3: Representative segment of error during test reference.

## References

- [1] Johan Kon, Dennis Bruijnen, Jeroen van de Wijdeven, Marcel Heertjes, Tom Oomen, "Physics-guided neural networks for feedforward control: an orthogonal projection-based approach," in *Proceedings of the American Control Conference, 2022* (to appear, [arXiv](https://arxiv.org/abs/2205.12345)).