Basis Function feedforward for Position-Dependent Systems

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Abstract

Feedforward for motion systems is getting increasingly more important to achieve performance requirements. This leads to a situation where position-dependent effects cannot be neglected anymore, where some examples can be seen in Figure 1.

Figure 1: Example position-dependent systems. Left: position-dependent flatbed printer. Center: position-dependent image guided therapy system. Right: timing-belt pulley system with position-dependent stiffness.

Feedforward for motion systems typically consists of basis functions and parameters, e.g., mass and snap feedforward,

$$u_{ff} = \mathrm{m}\frac{d^2}{dt^2}r + \delta\frac{d^4}{dt^4}r.$$

However, for position-dependent systems, feedforward should include position-dependent terms.

Approach: Learn position-dependent feedforward parameters in a different domain to achieve a less complex parameterization, i.e.,

$$u_{ff} = \frac{d^2}{dt^2}\left(\underbrace{m(\rho)}_{\theta_1(\rho)}\underbrace{1}_{\psi_1}r + \underbrace{\delta(\rho)}_{\theta_2(\rho)}\underbrace{\frac{d^2}{dt^2}}_{\psi_2}r\right).$$

The parameters are learned by applying recent advances in machine learning techniques, more specifically, kernel regularized system identification as seen in Pilonetto *et al.* [1] and Blanken *et al*. [2]. The kernel used specifies a prior on the feedforward parameter, for example smoothness or periodicity, as a function of position.

Example: Consider the timing-belt pulley system with position-dependent stiffness in Figure 2. This system represents the timing-belt pulley system of the 3D printer seen in the right of Figure 1.

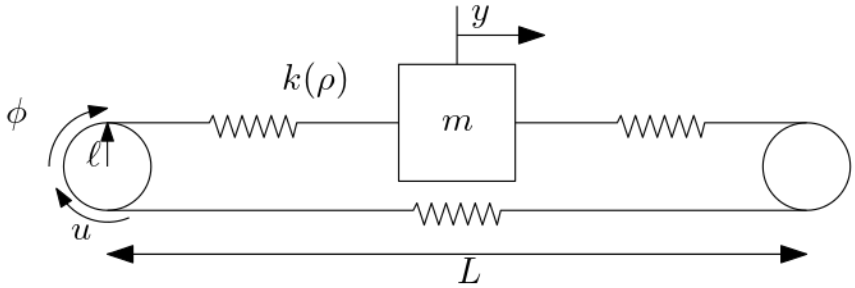

Figure 2: Timing-belt pulley system considered, with position-dependent spring stiffness $k(\rho)$.

The LPV input-output dynamics of this system, assuming one lumped spring with stiffness $k(\rho)$, are described by

$$\left(\frac{mJ}{r}\frac{d^2}{dt^2} + d\left(\frac{J}{r} + mr\right)\frac{d}{dt} + k(\rho)\left(\frac{J}{r} + mr\right)\right)y = \left(d\frac{d}{dt} + k(\rho)\right)\iint u\ dt^2.$$

Note that differentiation on both sides of this equation would lead to a complex expression due to the chain rule of differentiation, i.e., the operator $k(\rho)$ does not commute. The parameters are learned for the timing-belt pulley system with

$$k(\rho) = \frac{2L \cdot E \cdot A}{\rho\ell(2L - \rho\ell)} + 100 \cdot \sin(5 \cdot \rho),$$

where E is the elasticity modulus and A is the cross-sectional area of the timing belt. The developed approach that estimates the position-dependent snap feedforward parameter can be seen in Figure 3.

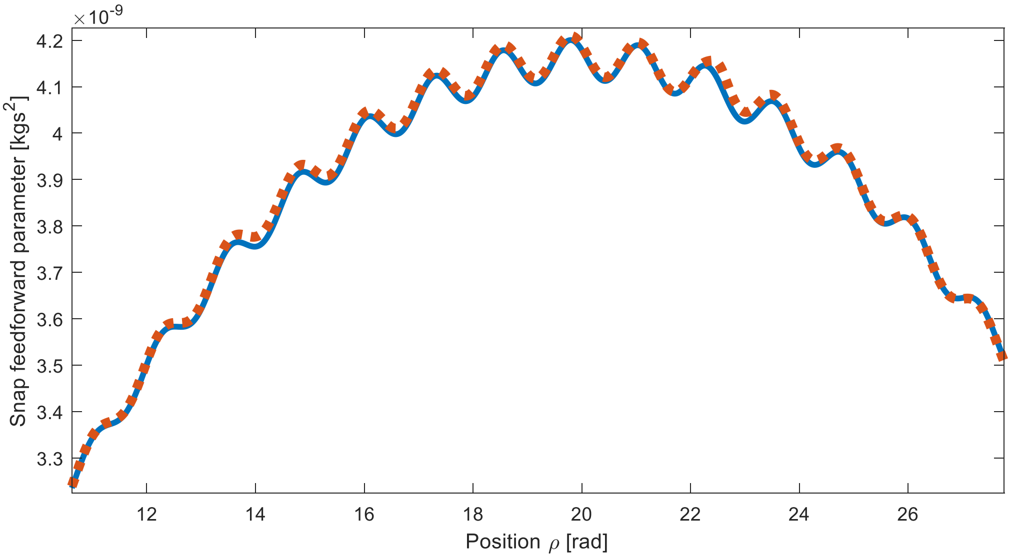

Figure 3: Learned (dotted orange) and true (solid blue) position-dependent snap feedforward parameter of the pulley system.

In terms of tracking performance, the normalized maximum error and error 2-norms are seen in Table 1.

Table 1. Normalized maximum tracking error and error 2-norm for a point-to-point tracking motion for the conventional position-independent feedforward and the developed approach.

Method	Maximum Error [-]	Error 2-norm [-]
Position-independent feedforward	1	1
Developed approach	0.024	0.027

References

[1] Pillonetto G, Dinuzz, F, Chen T, de Nicolao G and Ljung L 2014. Kernel methods in system identification, machine learning and function estimation: A survey *Automatica.* **50** 657-682

[2] Blanken L and Oomen T 2020. Kernel-based identification of non-causal systems with application to inverse model control *Automatica.* **114** 108830