Multirate Performance Quantification using Time-Lifting and Local Polynomial Modeling

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1 Background

Increasing performance requirements for mechatronic systems leads to continuous-time performance evaluation becoming more important. Continuous-time performance is typically evaluated using a significant higher sampling rate for the plant compared to the sampling rate of the controller, resulting in multirate systems. Similarly to single-rate systems, multirate systems require performance quantification.

2 Problem Formulation

Consider the multirate control structure in Figure 1, where the controller K_l is sampled at a low-rate $\omega_{s,l} = \omega_{s,h}/F$ and P_h at a high-rate $\omega_{s,h} = 2\pi/h_h$. Performance criteria

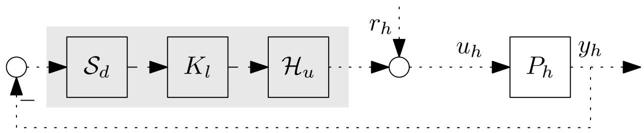

Figure 1: High-rate plant P_h operating in multirate feedback loop with low-rate controller K_l. The signals are up- and downsampled using $\mathcal{H}_u$ and $\mathcal{S}_d$.

for Linear Time Invariant (LTI) systems are typically evaluated using Frequency Response Functions (FRFs). However, since multirate systems are Linear Periodically Time-Varying (LPTV) [1], evaluating frequency domain models is not trivial. Several definitions for FRFs for multirate systems are available [2], requiring multiple experiments. Hence, the aim of this paper is to develop a more time-efficient method to identify multirate FRFs.

3 Approach

This paper considers the Performance Frequency Gain (PFG) definition of multirate FRFs, given by [2]

$$\mathcal{P}\left(e^{j\omega h_h}\right) = \sup_{w_h \neq 0} \frac{\|\zeta_h\|_{\mathcal{P}}}{\|w_h\|_{\mathcal{P}}}, \tag{1}$$

where ζ_h and w_h are chosen by the user, with w_h containing only one frequency component.The PFG represents the maximum response of a system for a single input frequency, hence requiring a multitude of experiments to determine for a frequency grid. An alternative representation of the PFG uses K_l and P_h [2]. To identify P_h, this paper uses the time-lifted representation, translating a SISO LPTV into a MIMO LTI representation [1]. First, define $\underline{J} : \underline{r} \mapsto \underline{y}$ and $\underline{S} : \underline{r} \mapsto \underline{u}$, where $\underline{r} = \mathcal{L}r_h$, $\underline{y} = \mathcal{L}y_h$, $\underline{u} = \mathcal{L}u_h$ and $\mathcal{L}$ the time-lifting operator. $\hat{\underline{J}}$ and $\hat{\underline{S}}$ are estimated with local polynomial modeling, since it identifies MIMO systems in a single experiment [3]. Second, recover the time-lifted original system as $\hat{\underline{P}} = \mathcal{L}\hat{P}_h\mathcal{L}^{-1} = \hat{\underline{J}}\,\hat{\underline{S}}^{-1}$. The high-rate system $\hat{P}_h$ is found by inverse lifting [1, Section 6.2.1]. Finally, the PFG is calculated by performing the procedure in [2], using $\hat{P}_h$ and K_l.

4 Initial Results

A high-rate fourth-order system P_h with controller K_l is considered with $F = 3$. The PFG is determined based on an estimate of P_h, both using an Emperical Transfer Function Estimate (ETFE) and the developed approach. The estimation error of the PFG for these methods is shown in Figure 2.

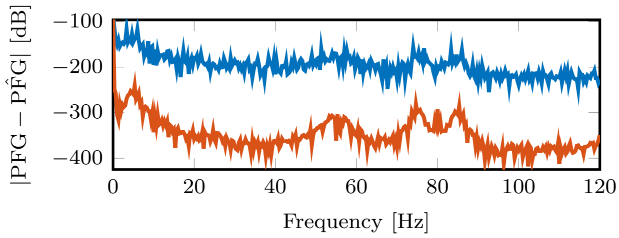

Figure 2: Error for estimating the PFG for ETFE (▬) and the developed approach (▬).

5 Ongoing Research

Ongoing research is focused at validating the framework in an experimental setting.

Acknowledgments

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References

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