Robust-control-relevant experiment design and system identification applied to a wafer stage

**Introduction**

Robust control
- Control goal
  \[ C_{\text{CRP}} = \arg \inf_{C} J_{\text{WC}}(P, C) \]
  \[ J_{\text{WC}}(P, C) = \| W T(P, C) V \|_{\infty} \]
- High performance requires small set \( P \)
- Set must be identified from data

Classical identification
- Delivers ellipsoidal uncertainty region \( \Theta \)
- Non-standard in robust control
- Central model non-optimal in view of \( J_{\text{WC}} \)

Experiment design
- Enables shaping \( \Theta \) via excitations \( w \)
- No straightforward connection to \( J_{\text{WC}} \)

**Goal:** Improve robust control performance by connecting robust control, model set identification, and experiment design:

\[ \{ C_{\text{CRP}}, P_{\text{RCR}}, w_{\text{RCR}} \} = \arg \inf_{C, P, w} J_{\text{WC}}(P(w), C) \]

**Method**

Control-relevant distance measure
- Control performance is bounded by
  \[ J(P, C) \leq J(\hat{P}, C) + \| W T(P, C) - T(\hat{P}, C) \|_{\infty} \]
- \( d \) is a control-relevant distance measure

Uncertainty structure selection
- Dual Youla-Kučera parametrization:
  \[ P = \{ \hat{P} : \hat{P} = (N + D_{\Delta})(D - N_{\Delta})^{-1} \}, \quad \| \Delta \|_{\infty} < \gamma \]
- Exploit non-normalized coprime factors \( \{ N, D \} \) and \( \{ N_{\Delta}, D_{\Delta} \} \) of \( \hat{P} \) and \( C_{\exp} \) to achieve robust-control-relevant model set:
  \[ d(P, \hat{P}) = \| G - \hat{G} \|_{\infty} = \| \Delta \|_{\infty}, \quad G = \begin{bmatrix} N & D \end{bmatrix} \]

Control-Relevant Coprime Factor Identification
Parametrization: \( G(\theta) \) by \( P(\theta) = B(\theta)A^{-1}(\theta) \)

Stage I: Non-parametric Chebicheff center

\[ G_{\text{CRP}} = \inf_{G \in C} \sup_{G \in D} d(G, \hat{G}) \]
Completely computed exactly by semi-definite program

Stage II: Parametric model

\[ G_{\text{CRP}} = \arg \inf_{G \in C, D} d(G_{\text{CRP}} - \hat{G})(\theta) \]
Guaranteed coprimeness

Control-Relevant Experiment Design
- Spectrum design \( \Phi_{w} \) that connects to identification method:
  \[ \Phi_{w_{\text{RCR}}} = \inf_{\Phi_{w} \in C} \sup_{G \in D} d(G, \hat{G}) \]
subject to experimental constraints

**Wafer Stage Results**

Control-Relevant Coprime Factor Identification

- Frequency Response Function Measurement
- Identified parametric model \( G(\theta) \)

Control-Relevant Experiment Design
- Uniform spectrum design: distance \( d \) is large at 3rd resonance
- Control-relevant spectrum design: distance \( d \) is reduced 4x by tailored excitations around control bandwidth and resonances

Control-Relevant Model set
- Control-relevance: tight around bandwidth and resonances, loose far below/beyond bandwidth

**References**