Closed-loop Aspects in MIMO Fault Diagnosis with Application to Precision Mechatronics

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Abstract—Fault detection is essential in precision mechatronics to facilitate maintenance and minimize operational downtime. The aim of this paper is to develop a systematic procedure from identification to accurate nullspace-based fault diagnosis, accounting for the influence of noise and interaction in multivariable closed-loop control configurations. The influence of noise and interaction on the model estimate and fault diagnosis system are investigated through the use of closed-loop operators and by means of an illustrative case study.

I. INTRODUCTION

Predictive maintenance is essential for future precision mechatronics and is enabled by a combination of fault diagnosis and digital twins [1]–[6]. The attention for predictive maintenance through fault diagnosis is mainly driven by the high cost associated with unscheduled downtime. Fault diagnosis is widely used in many application domains finding its origin in safety-critical domains such as aerospace, automotive and chemical industries, whereas this paper focuses on the domain of precision mechatronics. Fault diagnosis amounts to three tasks, that is, fault detection, fault isolation and fault identification.

Nullspace-based fault detection and isolation (FDI) methods have been developed [7]–[9] and enable fault diagnosis for large-scale complex systems. One of the main appeals of the nullspace-based approach is that it is generally applicable and is based on numerically reliable and computationally efficient techniques.

Model-based FDI methods, such as the nullspace-based method, rely on models of the underlying system dynamics. Two methods for modeling can be pursued: first principles modeling or data-driven modeling. A key question is how to obtain an accurate model which is suitable to serve as a basis for the fault diagnosis system. For precision mechatronics, data-driven modeling as opposed to first principles modeling, is fast, accurate, and inexpensive [10]–[12]. Moreover, these systems typically operate in closed-loop configuration, e.g., due to safety constraints. Closed-loop identification problems have been thoroughly analyzed in [13]. In particular, the non-parametric frequency response function-based (FRF) identification for multi-input multi-output (MIMO) complex motion systems has recently been investigated in [14]. Many methods exist to acquire accurate parametric models from FRFs [15], [16]. Despite the major development of identification for control, at present the identification of models for fault detection is missing.

Although important progress has been made in fault detection for complex engineered systems, at present closed-loop aspects in identification for fault diagnosis and the closed-loop aspects in fault diagnosis have not been clarified. The present research is driven by a lack of integral procedure for closed-loop controlled mechatronic systems. Hence, the main contribution of this paper is the development of a systematic procedure for fault diagnosis for MIMO systems in closed-loop configuration, starting from the identification of an accurate model. The relevance of this framework is highlighted through the investigation of closed-loop operators as well as an illustrative case study. An experimental case study, using an overactuated motion system, is described in [17].

II. PROBLEM FORMULATION

The operation of systems in closed-loop configuration has major implications for FDI and related system identification. In this paper, these implications are illustrated by answering the following questions.

i) How to accurately estimate a model for FDI despite presence of finite time noisy observations? (Sections III-A and III-B)

ii) What are the implications of MIMO and closed-loop aspects for model estimation for FDI? (Section III-B)

iii) How to minimize the impact of noise on the fault diagnosis system? (Section III-C)

iv) What are the implications of MIMO and closed-loop aspects for fault detection? (Section III-D)

The answers to these questions form a procedure for fault diagnosis design for MIMO systems, starting with the estimation of an accurate model. The main focus lies on the fault detection (FD) task, i.e., fault isolation and identification are not considered in the present paper.

First, the closed-loop setting for FRF estimation is introduced. Thereafter, the setting for FD is introduced.

A. Closed-loop identification problem

The closed-loop identification problem is considered as two-step approach. First, a non-parametric FRF is estimated. Thereafter, a parametric model is obtained through an appropriate fitting procedure. First, the setting for non-parametric FRF estimation is introduced. For the second step, the reader is referred to, e.g., [15], [16].

Consider the continuous time linear time-invariant (LTI) MIMO system described with transfer function matrix (TFM) \( G_u \) with input signal \( u : \mathbb{R}^+ \to \mathbb{R}^k \), defined at time \( t \in \mathbb{R}^+ \),
as depicted in Figure 1. The output trajectory \( y : \mathbb{R}_+ \rightarrow \mathbb{R}^m \) is given by

\[
y = v + G_u u,
\]

(1)

where and the additive zero-mean stationary stochastic noise contribution takes values \( v \in \mathbb{R}^m \) with spectral density \( \Phi_v \).

This can be formulated as noise model \( v = G_d d \), where \( d \in \mathbb{R}^p \) takes values from zero-mean white noise. In the closed-loop configuration, the output is equal to

\[
y = (I + G_u C)^{-1} G_u r + (I + G_u C)^{-1} v,
\]

(2)

and the input is given by

\[
u = (I + CG_u)^{-1} r - (I + CG_u)^{-1} Cv.
\]

(3)

The first goal of the closed-loop identification problem is to obtain a nonparametric model of \( G_u \) through the known input \( u \), the measured output \( y \) and the signal \( r : \mathbb{R}_+ \rightarrow \mathbb{R}^k \).

Depending on the application, also the controller \( C \) might be known. Note that by setting \( C = 0 \) and considering \( r = u \), the standard open loop identification problem is found [11], as highlighted in Figure 1.

**Assumption 1** It is assumed that in case the estimate \( \hat{G}_u(e^{j\omega}) \) is an accurate representation of \( G_u(e^{j\omega}) \), no additional modeling error is introduced through fitting procedures in order to obtain a parametric model.

**B. Closed-loop fault detection problem**

The configuration for closed-loop fault detection, illustrated in Figure 2, is highly similar to that used for system identification, cf. Figure 1. However, now additive faults \( f \in \mathbb{R}^q \) affect the output \( y \) through the TFM \( G_f(s) \). Hence, the output is given by

\[
y = (I + G_u C)^{-1} G_u r + (I + G_u C)^{-1} G_f f
\]

(4)

\[
+ (I + G_u C)^{-1} G_d d,
\]

and the input is given by

\[
u = (I + CG_u)^{-1} r - (I + CG_u)^{-1} CG_f f
\]

(5)

\[
- (I + CG_u)^{-1} CG_d d.
\]

Moreover, the system is augmented by a residual generator formed by a proper and stable TFM \( Q := [Q_y \ Q_u] \). The residual \( \varepsilon \), used for fault detection, is equal to

\[
\varepsilon = Q_u u + Q_y y.
\]

(6)

Loosely speaking, the fault detection goal is that the residual \( \varepsilon \neq 0 \), i.e., is sufficiently larger than zero in the presence of faults and the residual \( \varepsilon \approx 0 \), i.e., is sufficiently small in the absence of faults. Note that by setting \( C = 0 \) and considering \( r = u \), the standard open loop fault detection problem [8], where \( y \) is given by

\[
y = G_u u + G_f f + G_d d,
\]

(7)

is obtained as highlighted in Figure 2.

**C. Illustrative multivariable setup**

The system identification and fault detection problem are demonstrated on a representative model that consists of two DC motors that are interconnected by an elastic element as depicted in Figure 3, and illustrated in Figure 4. The elastic element enforces a strong cross-coupling between the two inputs and two outputs of the system. The system is represented by the state-space description

\[
\dot{x} = Ax + Bu,
\]

(8a)

\[
y = Cx,
\]

(8b)

with

\[
A = \begin{bmatrix}
-\frac{J_1 + J_2}{J_1 J_2} & \frac{J_1}{J_1 J_2} & \frac{J_2}{J_1 J_2} & \frac{J_1 + J_2}{J_1 J_2} \\
\frac{J_2}{J_1 J_2} & -\frac{J_1 + J_2}{J_1 J_2} & \frac{J_1}{J_1 J_2} & \frac{J_2}{J_1 J_2} \\
0 & \frac{J_1}{J_1 J_2} & -\frac{J_1}{J_1 J_2} & \frac{J_2}{J_1 J_2} \\
\frac{J_1}{J_1 J_2} & \frac{J_2}{J_1 J_2} & \frac{J_1}{J_1 J_2} & -\frac{J_2}{J_1 J_2}
\end{bmatrix},
B = \begin{bmatrix}
\frac{1}{J_1} & 0 & 0 \\
0 & 0 & \frac{1}{J_2} \\
0 & \frac{1}{J_1} & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

where the output \( y := [y_1 \ y_2]^\top \in \mathbb{R}^2 \) consists of the angular positions of both masses measured through optical encoders and the input \( u := [u_1 \ u_2]^\top \in \mathbb{R}^2 \) consists of the torques exerted by the motors. The state vector, denoted by \( x := [x_1 \ x_2 \ x_3 \ x_4] \in \mathbb{R}^4 \), corresponds to the angular positions \( x_1 \) and \( x_3 \), and angular velocities \( x_2 \) and \( x_4 \). The TFM of the system is equal to \( G_u(s) = C(sI - A)^{-1}B \). The multivariable system is controlled with a decentralized controller, i.e., a diagonal controller of the form \( C(s) := \text{diag}(C_{11}(s), C_{22}(s)) \). A decentralized controller for this system can, for instance, be designed through sequential loop closing.
III. PROCEDURE FOR CLOSED-LOOP FAULT DETECTION
BASIS ON IDENTIFIED MODELS

In this section, a procedure is presented for the closed-loop fault detection problem described in Section II-B based on identified models from the problem described in Section II-A. This is pursued systematically on the basis of answering the questions i) to iv) proposed in Section II.

A. Standard open loop identification solution

In this subsection, question i) from Section II is addressed. First, we consider a single-input single-output (SISO) system operating in open loop. The implications of considering a MIMO system and a closed-loop configuration are addressed in III-B.

To obtain an accurate estimate $\hat{G}_u(e^{j\omega})$ of the system $G_u(e^{j\omega})$, despite the presence of noise, consider one of the various commonly accepted solutions for the open loop problem. For arbitrary signals $u$ and $y$, i.e., generated through applying white noise input $u$, spectral analysis is a commonly applied solution. Here, after averaging over $i = N$ time-frames, the cross-power spectrum of the output and input, $\Phi_{yu}(\omega)$, is divided by the auto-power spectrum of the input, denoted by $\Phi_{uu}(\omega)$. This gives

$$\hat{G}_u(e^{j\omega}) = \frac{\Phi_{yu}(\omega)}{\Phi_{uu}(\omega)} = \frac{1}{N} \sum_{i=1}^{N} Y_i(\omega)U_i(\omega)H, \quad (9)$$

where $H$ denotes the Hermitian transpose and $Y_i(\omega)$ and $U_i(\omega)$ are the Fourier transforms of $y_i$ and $u_i$ corresponding to the $i^{th}$ frame. For smoothing purposes, typically a window, e.g., a Hamming window, is applied to each frame. Moreover, more frames may be created through overlap. This approach is often referred to as Welch’s modified periodogram method [18]. The coherence function can be considered as a quality certificate for the estimate. For estimates of the covariance on the transfer function estimate, the reader is referred to [19]. Despite noise entering the open loop system, Welch’s modified periodogram method provides an accurate solution to minimize the effect of noise so that an accurate FRF estimate is obtained.

B. MIMO and closed-loop implications on model estimate

In this subsection, question i) and ii) from Section II are addressed. Consider the closed-loop configuration from Figure 1, where a SISO system is excited through the signal $r$. Using the signals $u$ and $y$, similar to (9) gives the estimate [20, Chapter 10]

$$\hat{G}_u(e^{j\omega}) = \frac{\Phi_{yu}(\omega)}{\Phi_{uu}(\omega)} = \frac{G_u(e^{j\omega})\Phi_{rr}(\omega) - C^H(e^{j\omega})\Phi_{vv}(\omega)}{\Phi_{rr}(\omega) + |C(e^{j\omega})|^2 \Phi_{vv}(\omega)}.$$ \hspace{1cm} (10)

As a result of the correlation between $u$ and $v$, a biased estimate is obtained, which may lead to severe consequences for control and fault diagnosis system design. In the worst-case, if the system is solely driven by $v$, the obtained estimate is equal to the inverse controller.

A method to overcome this bias, is to estimate the FRF indirectly through the sensitivity function $S$ and the process sensitivity function $GS$. These estimates are given by

$$\hat{S}(e^{j\omega}) = \frac{\Phi_{ur}(\omega)}{\Phi_{rr}(\omega)} = \frac{1}{N} \sum_{i=1}^{N} U_i(\omega)R_i(\omega)H,$$ \hspace{1cm} (11)

and

$$\hat{G}_uS(e^{j\omega}) = \frac{\Phi_{yu}(\omega)}{\Phi_{uu}(\omega)} = \frac{1}{N} \sum_{i=1}^{N} Y_i(\omega)R_i(\omega)H,$$ \hspace{1cm} (12)

respectively. Similarly, to estimate the sensitivity and the process sensitivity, Welch’s modified periodogram method may be applied [18]. Then, the estimate of the plant $G_u$ may be obtained through

$$\hat{G}_u(e^{j\omega}) = \frac{\hat{G}_uS(e^{j\omega})}{\hat{S}(e^{j\omega})}. \quad (13)$$

For SISO systems, this provides an accurate estimate.

The approach for open loop systems through (9) can easily be extended to the MIMO case of the system $G_u$ with $k$ inputs by performing $k$ experiments, where each experiment independently excites one of the inputs in order to estimate a column of $G_u$. For alternative approaches, requiring a single experiment, see for instance [12, Chapter 7].

However, for closed-loop systems, the relation (13) has to be handled with care. To illustrate this, consider the MIMO configuration with two inputs and two outputs with a decentralized control configuration, depicted in Figure 5. The relation between input $u_1$ and output $y_1$ is now the equivalent plant defined as

$$G_{u,11}^{eq} := G_{u,11} - \frac{G_{u,12}C_{22}G_{u,21}}{1 + C_{22}G_{u,22}}.$$ \hspace{1cm} (14)$$

Similarly,

$$G_{u,22}^{eq} := G_{u,22} - \frac{G_{u,21}C_{11}G_{u,12}}{1 + C_{11}G_{u,11}},$$ \hspace{1cm} (15)
Fig. 5. MIMO closed-loop configuration with decentralized control architecture. The equivalent plants $G_{u,11}^e$ (●) and $G_{u,22}^e$ (○) are highlighted.

gives the equivalent plant between input $u_2$ and output $y_2$. Clearly, neglecting the interaction due to cross-coupling leads to a biased estimate. In fault diagnosis, working with an equivalent plant has its advantages. For instance, for systems with considerable amount of inputs and outputs, where the engineer is only interested in faults in a specific location or in case not all inputs and outputs are available to the fault diagnosis system.

If an estimate of the true MIMO plant is desired, $k$ independent excitation experiments have to be performed for each input. Each experiment allows to identify a column of the sensitivity estimate $\hat{S}(e^{j\omega})$ and process sensitivity estimate $\hat{G}_u S(e^{j\omega})$. Then, the estimate for the true MIMO plant can be obtained through the matrix product

$$\hat{G}_u(e^{j\omega}) = \hat{G}_u S(e^{j\omega}) \hat{S}(e^{j\omega})^{-1}.$$  

(16)

Even though the difference between (13) and (16) is subtle, the obtained estimate may be very different as a result of interaction.

Despite noise entering the closed-loop system during identification, there exist appropriate solutions to minimize its effect. However, the identification solution should carefully be selected such that biases do not enter the estimate on which the fault diagnosis system is designed, i.e., (13) for a SISO plant or the equivalent plants and (16) for the MIMO plant.

C. Open loop fault detection solution

In this subsection, question iii) from Section II is addressed. To design a linear residual generator $Q$ and minimize the impact of noise on the residual signal, consider the configuration from Figure 2. First, a MIMO system is considered in open loop configuration. Closed-loop aspects are addressed in Section III-D. The linear residual generator processing the measurable outputs $y$ and known inputs $u$ has input-output relation

$$\varepsilon = Q \begin{bmatrix} y \\ u \end{bmatrix}. \quad (17)$$

Substitution of the open loop relation for $y$, i.e., (4) with $C = 0$ and $r = u$ gives

$$\varepsilon = Q \begin{bmatrix} G_u & G_d \\ I & 0 \end{bmatrix} \begin{bmatrix} u \\ f \end{bmatrix}, \quad (18)$$

or equivalently

$$\varepsilon = (Q_u + Q_y G_u) u + (Q_y G_f) f + (Q_y G_d) d.$$  

(19)

The residual generator $Q$ can always be parameterized such that the residual $\varepsilon$ is decoupled from the input $u$ through the design criterion

a) $G_{eu} = 0$, which implies that $Q_u = -Q_y G_u$. The effects of the noise $d$ can usually not be decoupled from $\varepsilon$. Hence, $Q_y$ should be designed to achieve that the residual $\varepsilon$ is significantly influenced by all fault entries $f_i$, where $i = 1, \ldots, q$, and the influence of the noise signal $d$ is negligible. Thus, loosely speaking the two additional design conditions, which have to be fulfilled, are

b) $G_{ef} \neq 0$,
c) $G_{ed} \approx 0$.

Specifically, the objective is to maximize the gap between the fault detectability and noise attenuation. An optimization-based approach is employed following [9, Chapter 5]. Consider the admissible noise level $\gamma > 0$, imposed through

$$\|G_{ed}\|_\infty \leq \gamma; \quad (20)$$

where $\|\cdot\|_\infty$ denotes the $H_\infty$-norm. For $\gamma > 0$, a normalized value of $\gamma = 1$ can be used via suitable scaling of the residual filter $Q$. To characterize fault sensitivity, the index

$$\|G_{ef}\|_{\infty-} := \min_{1 \leq i \leq q} \|G_{efi}\|_\infty; \quad (21)$$

is used as a sensitivity measure, covering globally all fault inputs. Other indexes can be used, e.g., based on the least singular values of the frequency-response of $G_{ef}$ [21], [22]. Hence, given $\gamma \geq 0$, a stable and proper optimal fault detection filter $Q$ and corresponding optimal fault sensitivity level $\beta > 0$ need to be determined such that

$$\beta = \max_Q \left\{ \|G_{ef}\|_{\infty-} : \|G_{ed}\|_\infty \leq \gamma \right\}. \quad (22)$$

The gap $\frac{\beta}{\gamma}$ can be used as measure to assess the quality of the fault detection filter. Through coprime factorization, the poles of the residual generator can be assigned through which the speed of the FD process can be regulated.

In case the residual generator is based on an accurate fit of the non-parametric FRFs obtained through the methods described in Section III-B, the conditions a) to c) are accurately met. Moreover, the effect of noise during residual generation is minimized through the employed optimization-based approach.
D. MIMO and closed-loop implications on fault detection

In this subsection, question iv) from Section II is addressed. As described in Section II-B, closing the control loop has substantial consequences for model estimation. The impact on the fault diagnosis system investigated using closed-loop operators.

**Theorem 1** Let $G_u$, $G_d$, and $G_f$ be given transfer function matrices for the system described by (7) with residual generator (6), shown in Figure 2 (•) and let $G_u$, $G_d$, and $G_f$ be the same transfer function matrices for the system described by (4) with residual generator (6), shown in Figure 2. Then, decoupling the residual $\varepsilon$ from $u$ and maximizing (22) for the open loop configuration leads to the exact same residual generator $Q$ as decoupling the residual $\varepsilon$ from $v$ and maximizing (22) for the closed-loop configuration. Hence, the residual generation problem is invariant to the feedback-control loop, i.e., invariant to the controller $C$

Hence, the design criteria for the closed-loop configuration are the exact same as a) to c) in the open loop setting. The proof will be published elsewhere.

This result implies that when the entire MIMO plant $G_u$ is known, the same residual generator can be used in an open loop as well as in a closed-loop configuration.

In case the equivalent plant is used for fault diagnosis, e.g., because either the full MIMO plant is unknown or since it is of large dimensions, the fault detection problem is not invariant to the controller located in the equivalent plant.

Moreover, this result shows that despite the controller attenuating the effect of additive faults, it is equally detectable in the residual, which can be considered as a nice property. Indeed, the residual is thus not a metric that describes the severity of the fault in regard to the performance of the closed-loop system.

**IV. ILLUSTRATIVE CASE STUDY**

Through this simulation-based case study, the implications of closed-loop feedback on different fault detection filter designs are illustrated. The first filter is designed neglecting the influence of the noise $d$, the second filter is based on the assumption that interaction may be neglected, and the third filter takes both into account.

Consider the data-generating system described by (8) with $J_1 = J_2 = 0.02$, $k_1 = k_3 = 0.01$, $d_1 = d_3 = 0.02$, $k_2 = 500$ and $d_2 = 0.5$ with transfer function matrix $G_u(s)$ and decentralized controller as depicted in Figure 5, with $C_{11}(s) = \frac{550s + 14110}{s}$ and $C_{22}(s) = 1.75s + 110$. Moreover, $G_{d_1}(s) = G_{d_2}(s) = \frac{s}{s + 600\pi}$, representing high frequency noise with $d_1, d_2 \sim \mathcal{N}(0, 10^{-2})$.

First, the system dynamics are identified through two independent identical experiments with $r_1, r_2 \sim \mathcal{N}(0, 1)$, of which the result is depicted in Figure 6. The $2 \times 2$ MIMO plant is estimated with (16) and the equivalent plants are estimated with (13). Moreover, the bias obtained through estimating with (10) is illustrated through comparison with the true equivalent plants.

Fig. 6. Estimation of the MIMO transfer function matrix and equivalent plants, compared to the true $G_u$ (--) and the true equivalent plants $\hat{G}_{u,11}$ (---) and $\hat{G}_{u,22}$ (---). The MIMO estimate $\hat{G}_u$ (—) is indirectly identified with matrix wise multiplication of the process sensitivity and inverse sensitivity. The equivalent plant estimates $\hat{G}_{u,11}$ (—) and $\hat{G}_{u,22}$ (—) are estimated via element-wise division of the process sensitivity and inverse sensitivity. Moreover, the biased estimate (---), identified through the use of solely $u$ and $y$ is depicted.

**Assumption 2** It is assumed that the estimates $\hat{G}_u$, $\hat{G}_{u,11}$, $\hat{G}_{u,22}$ are fit such that the true plants $G_u$, $G_{u,11}$ and $G_{u,22}$ are obtained for the following fault detection case study.

The latter is assumed in order to separate the effect of noise during model estimation and during fault detection for a fair comparison of the filter designs. Hence, the effect of biases as a result of model estimation are negated.

**Assumption 3** It is assumed that only the first actuator is sensitive to faults. Hence, it is assumed that only additive faults can enter the system at $u_1$, shown in Figure 5.

Since only faults enter the system at $u_1$, there is no direct interest in the remaining part of the system, i.e., only the equivalent plant $G_{u,11}$ as depicted in Figure 5 is considered.

Another motivation to work with this equivalent plant might be that $u_2$ and $y_2$ are not directly available for residual generation. Considering $G_{u,11}$, the actuator fault enters $y_1$ through $G_f = G_{u,11}^e$. Note that the disturbance $d_2$ enters $y_1$ through

$$
G_{d_2}^e := -\frac{G_{u,12}C_{22}}{1 + C_{22}G_{u,22}}G_{d_2}.
$$

The first mass is required to follow a sinusoidal setpoint, implemented through the signal $r_1 = C_{11}r_1$ with $r_1(t) = \sin(0.2\pi t)$. In the remainder, three residual generators are compared on the basis of different assumptions.

(1) Neglecting the influence of noise, i.e., based on $G_{u,11}^e$, $G_f = G_{u,11}^e$ and assuming $G_d = 0$.

(2) Neglecting interaction, i.e., based on $G_{u,11}$, $G_f = G_{u,11}$ and $G_{d_1}$.

(3) Including interaction, i.e., based on $G_{u,11}^e$, $G_f = G_{u,11}^e$, $G_{d_1}$ and $G_{d_2}^e$.

The considered additive fault is depicted in Figure 7, as well as the inputs, outputs and reference signal. The normalized residual signals for each of the filter designs are depicted in Figure 8. The residual signal of the first design (—)
illustrative case study, it is shown that care should be taken with respect to interacting submodules. The presented framework provides a solid basis for fault detection, predictive maintenance and digital twins for complex mechatronic systems.

V. CONCLUSION

The presented identification and fault detection framework in this paper addresses major ambiguities in closed-loop systems. This allows for fault detection in complex systems such as precision mechatronics where only submodules are identified and used in fault diagnosis system. Through an

### References