52nd IEEE Conference on Decision and Control
December 10-13, 2013. Florence, Italy

Iterative Feedforward Control: A Closed-loop Identification Problem and a Solution

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Abstract— **Feedforward control can significantly improve the performance of a system through compensation of disturbances. By exploiting measured data from previous tasks and a suitable feedforward parametrization, iterative feedforward control simultaneously attains high performance and good extrapolability of tasks. This paper aims to show that earlier contributions in this area suffer from a closed-loop identification problem. A novel solution is presented based on closed-loop identification techniques, which shows that existing feedforward control algorithms can be significantly enhanced. A simulation example confirms the existence of a closed-loop identification problem in earlier approaches and shows that the proposed solution is superior compared to pre-existing results.**

I. INTRODUCTION

Feedforward control is widely used in control systems, since feedforward can effectively reject disturbances before these affect the system. Indeed, many applications to high-performance systems have been reported where feedforward control leads to a significant performance improvement. For servo systems, the main performance improvement is in general obtained by using feedforward with respect to the reference signal. Relevant examples of feedforward control include model-based feedforward [1], [2] and Iterative Learning Control (ILC) [3].

On the one hand, model-based feedforward results in general in good performance and provides extrapolability of tasks. In model-based feedforward, a parametric model is determined that approximates the inverse of the system. The performance improvement induced by model-based feedforward is highly dependent on i) the model quality of the parametric model and ii) the accuracy of the model-inversion [4]. On the other hand, ILC results in superior performance with respect to model-based feedforward, at the expense of poor extrapolability of tasks. By learning from previous iterations, high performance is obtained for a single, specific task. In addition, ILC only requires an approximate model.

Recently, an approach is presented in [5] that combines the advantages of model-based feedforward and ILC, resulting in both high performance and good extrapolability properties. Thereto, basis functions are introduced that reflect the dynamical behavior of the system responsible for the dominant contribution to the servo error. In [6], the need for an approximate model of the system, as is common in ILC, is eliminated by exploiting concepts from iterative feedback tuning (IFT) [7]. This approach is extended to input shaping [8] and multivariable systems [9], while a comparative study of data-driven feedforward control procedures is reported in [10]. However, by eliminating the need for an approximate model of the system, the iterative feedforward tuning approach presented in [6] requires two tasks to perform an update of the feedforward controller.

Although iterative feedforward tuning is widely successful to improve the performance of a system, existing tuning procedures i) impose stringent requirements on noise acting on the system and ii) require two tasks for each iterative update of the feedforward controller. In this paper it is shown that both deficiencies can be removed by connecting iterative feedforward tuning to system identification, and exploit concepts from closed-loop system identification in iterative feedforward tuning.

The main contribution of this paper is the formulation of an iterative feedforward tuning procedure that is efficient, i.e., exploits measurements from a single task, while attaining typical performance requirements for feedforward control in the presence of noise. As a result, the deficiencies of the tuning procedures used in [6], [8] and [9] are resolved, i.e., a single task is sufficient to determine an iterative update of the feedforward controller in the presence of noise. The proposed approach is closely related to [11], [12] and [13], and extends this work to iterative tuning of feedforward controllers.

Notation. The indeterminate ξ is used to represent either s, z, or q for the continuous time, discrete time, and forward time shift case, respectively. Let $\mathbb{R}[\xi]^{n_1 \times n_2}$ denote the set of real polynomial matrices with n_1 rows and n_2 columns. For a vector x, $||x||_W = x^T W x$. Furthermore, the expected value $\mathbb{E}(x)$ is defined as $\mathbb{E}(x) = \int_{-\infty}^{\infty} x f(x) dx$, with probability density function $f(x)$. The correlation function based on a finite number of samples N is defined as $R_{xy} = \sum_{t=1}^{N} x(t)y(t)$. The asymptotic sample mean $\bar{\theta}$ and sample normalized variance $\hat{P}$ for m realizations are defined as $\bar{\theta} = \frac{1}{m}\sum_{l=1}^{m} \hat{\theta}_{N,l}$ and $\hat{P} = \frac{N}{m}\sum_{l=1}^{m}(\hat{\theta}_{N,l} - \bar{\theta})(\hat{\theta}_{N,l} - \bar{\theta})^T$, where $\hat{\theta}_{N,l}$ is the estimate from the l^{th} realization and N is the number of data points in each realization.

II. PROBLEM FORMULATION

A. Feedforward Control Goal

The goal in feedforward control is to improve performance by compensating for known exogeneous input signals that affect the system. Consider the two degree-of-freedom control configuration as depicted in Fig. 1. The true unknown system P is assumed to be discrete-time, single-input single-output, and linear time-invariant. The control configuration

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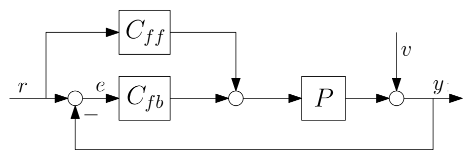

Fig. 1. Two degree-of-freedom control configuration

consists of a given stabilizing feedback controller C_{fb} and feedforward controller C_{ff}. Furthermore, let r denote the known reference signal, v the disturbance, and e the servo error. The disturbance v is modeled as $v = H\epsilon$, where H is monic and ϵ is zero mean white noise with variance λ_e^2. Hence, v and r are uncorrelated.

In iterative feedforward tuning, measurement data is exploited to update C_{ff} after each task. This data-driven feedforward optimization problem is formulated in Def. 1.

Definition 1. *Given measured signals e^j and y^j obtained during the j^{th} task of the closed-loop system in Fig. 1 with C_{ff}^j implemented. Then,*

$$C_{ff}^{j+1} = C_{ff}^j + C_{ff}^{\Delta}, \tag{1}$$

where the update C_{ff}^{Δ} based on e^j and y^j results from the optimization problem

$$C_{ff}^{\Delta,\, opt} = \arg \min_{C_{ff}^{\Delta} \in \mathcal{C}} V(C_{ff}^{\Delta}), \tag{2}$$

with criterion $V(C_{ff}^{\Delta})$ and feedforward controller parametrization $\mathcal{C}$.

The formulation of $V(C_{ff}^{\Delta})$ and $\mathcal{C}$ is essential for the performance of the system with C_{ff}^{j+1}. In this paper, a fixed parametrization $\mathcal{C}$ is adopted that encompasses common parametrizations in feedforward control, including those in [14], [6], [9] and [8].

Definition 2. *The feedforward controller C_{ff} is parametrized as*

$$\mathcal{C} = \left\{ C_{ff}(\xi,\theta) | C_{ff}(\xi,\theta) = \sum_{k=1}^{n_\theta} \psi_i \theta_i,\ \psi_i \in \mathbb{R}[\xi], \theta_i \in \mathbb{R} \right\},$$

with parameter vector

$$\theta = [\theta_1, \theta_2, \ldots, \theta_{n_\theta}]^T \in \mathbb{R}^{n_\theta \times 1}, \tag{3}$$

and basis functions $\Psi = [\psi_1, \psi_2, \ldots, \psi_{n_\theta}] \in \mathbb{R}[\xi]^{1\times n_\theta}$.

B. Problem Formulation and Outline

In iterative feedforward tuning, the performance of the system in Fig. 1 is improved by exploiting measurements from the previous tasks to iteratively update C_{ff}. As stated in (2), the update $C_{ff}^{\Delta} \in \mathcal{C}$ is determined by $V(C_{ff}^{\Delta})$. In fact, two key requirements for $V(C_{ff}^{\Delta})$ in iterative feedforward tuning are identified:

R1. Minimization of the experimental cost, such that C_{ff} can be updated after each task.

R2. Estimation of (3) in C_{ff}^{Δ} such that the error e induced by r is minimized, despite the presence of v in the performed task.

In view of the identified requirements R1–R2 for $V(C_{ff}^{\Delta})$, the contribution of this paper is twofold. First, an analysis is provided of existing iterative feedforward tuning procedures. It is shown that in these approaches, R1 and R2 are conflicting in the presence of v. This paper reveals that the underlying problem can be interpreted as a closed-loop identification problem. Second, a novel formulation for $V(C_{ff}^{\Delta})$ is proposed that attains requirements R1–R2. This is achieved by establishing a novel connection to closed-loop identification techniques.

This paper is organized as follows. In Section III, it is shown that existing criteria provide an analytic solution θ to (2). For clarity of exposition, attention is restricted to a single update of C_{ff}. In Section IV, this analytic expression for θ is exploited to show that in the presence of v, existing criteria suffer from a closed-loop identification problem. In Section V, a novel optimization criterion is provided that is inspired by closed-loop identification techniques. In Section VI, the proposed approach is embedded in the iterative feedforward tuning framework. An example confirming the claims is provided in Section VII. Finally, conclusions are provided in Section VIII.

III. NOISE-FREE SOLUTION

In this section, the data-driven approach discussed in [6], [9], and [8] is presented for estimating the parameter vector θ for the feedforward controller $C_{ff}(q,\theta) \in \mathcal{C}$. In this approach, the two-norm of the measured error signal e is minimized, under the assumption that the disturbance v is negligible. For clarity of exposition, it is assumed that C_{ff}^{j+1} in (1) is equal to C_{ff}^{Δ}, i.e., C_{ff}^{j+1} is determined based on a single task without prior feedforward controller.

Definition 3. *The criterion in* (2) *is defined as*

$$V_2(\theta) = ||e(\theta)||_2^2, \tag{4}$$

where $e(\theta)$ is given by

$$e(\theta) = S(1 - PC_{ff}(q,\theta))r - Sv. \tag{5}$$

Crucially, θ should be estimated based on measurement data only in a data-driven approach, i.e., without explicitly constructing parametric or nonparametric models of closed-loop transfer functions, see, e.g., [15].

The measured signals $e_m(t)$ and $y_m(t)$, contaminated by the disturbance v acting on the closed-loop system, are given by

$$e_m(t) = e_r(t) + e_v(t) = Sr(t) - Sv(t),$$
$$y_m(t) = y_r(t) + y_v(t) = SPC_{fb}r(t) + Sv(t).$$

The following result is essential for subsequent derivations.

Lemma 1. *Assume that $v(t) = 0$ for $t = 1, ..., N$. Then, the mapping from $r(t)$ to $y(t)$ given by*

$$y = SPC_{fb}r,$$

is equivalent to

$$SPr = C_{fb}^{-1} y. \tag{6}$$

A proof follows along the same lines as in [6]. As shown in the following theorem, this auxiliary result enables the estimation of θ without the use of a model of SP. Consequently, $C_{ff}(q, \theta)$ can directly be estimated from measurement data.

Theorem 1. *Given measured signals $e_r(t)$, $y_r(t)$ for $t = 1, ..., N$ and suppose that $v(t) = 0$ $\forall t$. Then, for $C_{ff} \in \mathcal{C}$, minimization of* (4) *with respect to $\hat{\theta}_N$*

$$\hat{\theta}_N = \arg\min_{\theta} V_2(\theta), \tag{7}$$

is equivalent to the least squares solution to

$$\Phi \hat{\theta}_N = e_r, \tag{8}$$

with $\Phi(q) = \Psi(q) C_{fb}^{-1}(q) y_r \in \mathbb{R}^{N \times n_\theta}$.

Proof. Suppose that $v(t) = 0$ $\forall t$. Then, (5) is equal to

$$e(\theta) = e_r - SPC_{ff} r. \tag{9}$$

Substitution of (6) in (9) yields

$$\begin{aligned} e(\theta) &= e_r - C_{ff}(q, \theta) C_{fb}^{-1} y_r, \\ &= e_r - \Phi(q)\theta. \end{aligned}$$

Since $C_{ff}(q, \theta) \in \mathcal{C}$ is linear in θ and $V(\theta)$ is a positive-definite function, $\hat{\theta}_N$ is the unique solution to

$$\frac{\partial V_2(\theta)}{\partial \theta}\Big|_{\theta = \hat{\theta}_N} = 0,$$

resulting in the linear regression problem as formulated in (8). □

Remark 1. *In [6, Section 2.4], nonlinear optimization is used to solve* (7)*. Inspired by iterative feedback tuning (IFT) [7], this procedure to iteratively update C_{ff} relies on approximations of the Hessian and gradient of* (4)*, resulting in an estimate of θ. However, Thm. 1 shows that the optimization problem* (7) *has an analytic solution.*

The least squares solution to (8) is equivalent to

$$\hat{\theta}_N = (\Phi^T \Phi)^{-1} \Phi^T e_r.$$

The following assumption ensures that $\hat{\theta}_N$ can be uniquely computed.

Assumption 1. *$\Phi^T \Phi$ is nonsingular.*

Assumption 1 imposes a persistence of excitation condition on r. For $C_{ff}(\xi, \theta)$ as in Def 2, the number of parameters that are uniquely determined is equal to the order of persistence of excitation of the signal r. Since n_θ is typically small, Assumption 1 is nonrestrictive. For the purpose of analysis in the next section, consider the following definition of the optimal feedforward controller.

Definition 4. *The optimal feedforward controller with respect to* (4) *is defined as $C_{ff}^{opt}(q, \theta_N^*) \in \mathcal{C}$ where θ_N^* is the least squares solution to* (8) *for noise-free measurements.*

Remark 2. *The preview-based stable inversion approach in [16] is used to compute $C_{fb}^{-1} y$ for unstable C_{fb}^{-1}.*

The key question addressed in the next section is whether the least squares solution $\hat{\theta}_N$ to (8) is an unbiased estimate of θ_N^* for noisy measurements of e and y.

IV. ANALYSIS IN THE PRESENCE OF NOISE

In this section, the properties of the data-driven approach in Section III are analyzed in the presence of the disturbance v. It is shown that the presence of v in the closed-loop system depicted in Fig. 1 results in a biased estimator $\hat{\theta}_N$ for θ_N^* of $V_2(\theta)$, due to a closed-loop identification problem.

Following a similar reasoning as in Lemma 1, (6) equals

$$(SPr)_{\text{est}} = C_{fb}^{-1} y_m. \tag{10}$$

By evaluating its expected value,

$$\mathbb{E}(SPr)_{est} = \mathbb{E}\left\{C_{fb}^{-1}\left[y_r + y_v\right]\right\} = SPr.$$

From (10) it follows that the approximation of SPr is unbiased and hence seems suitable. In the remainder of this section, the implications of (10) are analyzed for the estimation of θ. First, the optimization problem with respect to θ in the presence of v is stated in the following proposition.

Definition 5. *Given measured signals $e_m(t)$, $y_m(t)$ for $t = 1, ..., N$. Then, $\hat{\theta}_N$ is defined as the least squares solution to*

$$\hat{\Phi} \hat{\theta}_N = e_m, \tag{11}$$

where $\hat{\Phi} = \Psi(q) C_{fb}^{-1}(q) y_m \in \mathbb{R}^{N \times n_\theta}$.

Next, the properties of $\hat{\theta}_N$ in Def. 5 are analyzed. Define $\hat{\Phi} = \Phi + \Phi_v$, where $\Phi = [\varphi_1, \varphi_2, \ldots, \varphi_{n_\theta}]$ is the noise-free part and $\Phi_v = [\varphi_1^v, \varphi_2^v, \ldots, \varphi_{n_\theta}^v] = \Psi(q) C_{fb}^{-1}(q) y_v$. The least squares solution to (11) is given by

$$\hat{\theta}_N = (\hat{\Phi}^T \hat{\Phi})^{-1} \hat{\Phi}^T \left[e_r + e_v\right], \tag{12}$$

where

$$\begin{aligned} \hat{\Phi}^T \hat{\Phi} &= \Phi^T \Phi + F_1 + B_1, \\ \hat{\Phi}^T \left[e_r + e_v\right] &= \Phi^T e_r + F_2 + B_2. \end{aligned}$$

The matrices F_1, B_1, F_2 and B_2 are given by

$$\begin{aligned} F_1 &= \Phi^T \Phi_v + \Phi_v^T \Phi, \quad & B_1 &= \Phi_v^T \Phi_v, \\ F_2 &= \Phi^T e_v + \Phi_v^T e_r, \quad & B_2 &= \Phi_v^T e_v. \end{aligned}$$

The entries of F_1, F_2, B_1 and B_2 are correlation functions based on a finite number of samples N. First, F_1 and F_2 are considered. The entries of F_1 and F_2 are equal to the cross-correlation functions of the filtered signals v and r. Recall from Section II-A that v is uncorrelated with the reference signal r. This leads to the following result.

Lemma 2. *$\mathbb{E}F_1(i, k) = 0$, $\mathbb{E}F_2(i) = 0$ $\forall i, k$.*

Proof. $\mathbb{E}F_1(i, k)$ and $\mathbb{E}F_2(i)$ are given by

$$\begin{aligned} \mathbb{E}F_1(i, k) &= \mathbb{E}R_{\varphi_i \varphi_j^v} + \mathbb{E}R_{\varphi_i^v \varphi_j}, \\ \mathbb{E}F_2(i) &= \mathbb{E}R_{\varphi_i e_v} + \mathbb{E}R_{\varphi_i^v e_r}. \end{aligned}$$

For v uncorrelated with the reference signal r, the following conditions hold $\forall i, j$:

$$\mathbb{E}R_{\varphi_i \varphi_j^v} = 0, \quad \mathbb{E}R_{\varphi_i^v e_r} = 0,$$
$$\mathbb{E}R_{\varphi_i^v \varphi_j} = 0, \quad \mathbb{E}R_{\varphi_i e_v} = 0.$$

implying that $\mathbb{E}F_1(i,k) = 0$, $\mathbb{E}F_2(i) = 0$ $\forall i, k$. □

This result shows that F_1 and F_2 do not contribute to $\mathbb{E}(\hat{\theta}_N)$. The following lemma reveals that B_1 and B_2 are nonzero and have an effect on $\mathbb{E}(\hat{\theta}_N)$.

Lemma 3. *$\mathbb{E}B_1(i,k) > 0$, $\mathbb{E}B_2(i) > 0$ $\forall i, k$ for $\lambda_\epsilon > 0$.*

This result follows by observing that the entries of B_1 and B_2 are (filtered) auto-correlation functions of v. Combining Lemma 2 and Lemma 3 leads to the following result, that holds for finite and infinite N.

Theorem 2. *Given the measured signals $e_m(t)$ and $y_m(t)$ for $t = 1, \ldots, N$. Then, $\mathbb{E}\hat{\theta}_N \neq \theta_N^*$ for $\lambda_\epsilon > 0$.*

Proof. The expected value of (12) is given by

$$\mathbb{E}\hat{\theta}_N = (\Phi^T \Phi + B_1)^{-1} \left[\Phi^T e_r + B_2\right].$$

Lemma 3 implies that $\mathbb{E}\hat{\theta}_N \neq \theta_N^*$ for $\lambda_\epsilon > 0$. □

Summarizing, the approach presented in [6], [9], and [8], based on minimizing $V_2(\theta)$, results in $\mathbb{E}\hat{\theta}_N \neq \theta_N^*$ for $\lambda_\epsilon > 0$, when measurements from a single task are used. This shows that the requirements R1 and R2 in Sect. II-B are conflicting for $V_2(\theta)$.

Remark 3. *In [6, Section 2.4], inspired by a similar approach developed in IFT [7], a procedure is proposed that results in $\mathbb{E}\hat{\theta}_N = \theta_N^*$, at the expense of measuring two tasks. However, this two-step approach implies that C_{ff} can not be updated after each task, thereby violating requirement R1.*

V. SOLUTION BASED ON INSTRUMENTAL VARIABLES

In this section, a novel procedure is presented that exploits knowledge of r in $V(\theta)$ to simultaneously achieve requirements R1–R2 in Sect. II-B. A key contribution of this paper is to show that the closed-loop identification problem that exists in (12) due to the contribution of v is eliminated by explicitly using r in the optimization problem in (2). To this purpose, a connection is proposed between instrumental variable identification techniques [11] and iterative feedforward tuning [6]. For iterative feedforward tuning, the corresponding criterion is posed in the following definition.

Definition 6. *The criterion in* (2) *is defined as*

$$V_z(\theta) = \left|\left|Z^T e(\theta)\right|\right|_W^2, \tag{13}$$

where $Z \in \mathbb{R}^{N \times n_z}$ are instrumental variables, W is a positive-definite weighting matrix, and $n_z \geq n_\theta$.

In this section, the basis instrumental variable approach is pursued, see, e.g., [11, Chapter 3], in which case $Z \in \mathbb{R}^{N \times n_\theta}$ and $W = I \in \mathbb{R}^{N \times N}$. The parameters θ of $C_{ff}(q, \theta)$ then result from the set of equations

$$Z^T \left[e_m - \hat{\Phi}\hat{\theta}_N^{IV}\right] = 0. \tag{14}$$

The solution to (14) is given by

$$\hat{\theta}_N^{IV} = (Z^T \hat{\Phi})^{-1} Z^T e_m, \tag{15}$$

where

$$Z^T \hat{\Phi} = Z^T \Phi + S_1$$
$$Z^T e_m = Z^T e_r + S_2,$$

with $S_1 = Z^T \Phi_v$ and $S_2 = Z^T e_v$. The following assumption guarantees that $\hat{\theta}_N^{IV}$ can be uniquely determined.

Assumption 2. *$Z^T \hat{\Phi}$ is nonsingular.*

Assumption 2 implies that Z should be correlated with $\hat{\Phi}$. For noise-free measurements of e_m and y_m, $\hat{\theta}_N^{IV}$ yields

$$\hat{\theta}_N^{IV} = (Z^T \Phi)^{-1} Z^T e_r, \tag{16}$$

leading to the following definition for C_{ff}^{opt} based on (13).

Definition 7. *The optimal feedforward controller with respect to* (13) *is defined as $C_{ff}^{opt}(q, \theta_N^{IV,*}) \in \mathcal{C}$ where $\theta_N^{IV,*}$ is the solution to* (14) *for noise-free measurements.*

In contrast to (12), the freedom that exist in the construction of Z can be exploited to eliminate (filtered) auto-correlation functions of v from (15). This is illustrated in the remainder of this section. To proceed, define $Z = [\zeta_1, \zeta_2, \ldots, \zeta_{n_\theta}]$. The entries of S_1 and S_2 are cross-correlation functions of ζ_i and v. If ζ_i and v are uncorrelated $\forall i$, v is eliminated from (15). To this purpose, filtered values of r are used as instrumental variables.

Lemma 4. *Let Z consist of filtered values of r. Then, $\mathbb{E}S_1(i,k) = 0$, $\mathbb{E}S_2(i) = 0$, $\forall i, k$.*

Proof. Since v is uncorrelated with r,

$$\mathbb{E}S_1(i,k) = \mathbb{E}R_{\zeta_i \varphi_j^v} = 0 \quad \forall i, j,$$
$$\mathbb{E}S_2(i) = \mathbb{E}R_{\zeta_i e_v} = 0 \quad \forall i.$$

□

The following result follows from Lemma 4.

Theorem 3. *Given the measured signals $e_m(t)$ and $y_m(t)$ for $t = 1, \ldots, N$. Then, for Z as in Lemma 4, $\mathbb{E}\hat{\theta}_N^{IV} = \theta_N^{IV,*}$.*

Proof. The expected value of (15) is given by

$$\mathbb{E}\hat{\theta}_N^{IV} = (Z^T \Phi)^{-1} Z^T e_r,$$

which is equal to (16). □

Thm. 3 is valid for finite and infinite N. Note that there still exists freedom in the construction of Z if Assumption 2 and Lemma 4 are achieved. For $C_{ff} \in \mathcal{C}$ with basis functions Ψ, a typical design choice is to select $Z = [\psi_1 r, \psi_2 r, \ldots, \psi_{n_\theta} r]$.

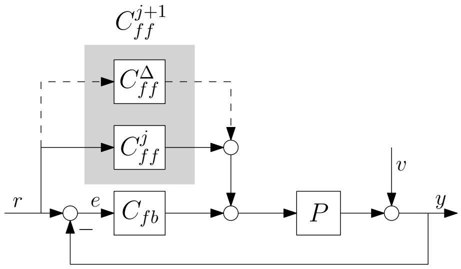

Fig. 2. Superposition of C_{ff}^{j+1}.

Concluding, a criterion $V_z(\theta)$ is formulated that exploits knowledge of r by establishing a novel connection between feedforward tuning and closed-loop identification techniques. As a result, measurements from a single task are sufficient to obtain $\mathbb{E}\hat{\theta}_N^{IV} = \theta_N^{IV,*}$. This illustrates that requirements R1–R2 in Sect. II-B are attained for $V_z(\theta)$.

VI. ITERATIVE TASKS

In this section, the instrumental variable method in Sect. V is embedded in the iterative task framework formulated in Sect. II. The main contribution of this paper is the formulation of a systematic procedure to improve the performance of the system in Fig. 1 by iteratively updating $C_{ff}(q,\theta)$. To this purpose, measurement data is exploited that is obtained in the previous task.

The pursued approach to adapt $C_{ff}(q,\theta)$ is to use recursive estimates of θ. Consider the two degree-of-freedom control configuration as depicted in Fig. 2. The feedforward controller $C_{ff}^{j}(q,\theta_N^j)$ in the j^{th} experiment is updated by $C_{ff}^{\Delta}(q,\hat{\theta}_N^{\Delta})$. Herein, $\hat{\theta}_N^{\Delta}$ is determined based on e_m^j and y_m^j in the j^{th} iteration given by

$$e_m^j = S(1-PC_{ff}^j)r - Sv,$$
$$y_m^j = SP(C_{fb}+C_{ff}^j)r + Sv.$$

Proposition 1. *Given e_m^j and y_m^j and $C_{ff}^j \in \mathcal{C}$ in the j^{th} iteration. The IV estimate $\hat{\theta}_N^{\Delta,\,IV}$ is the solution to*

$$\hat{\theta}_N^{\Delta} = (Z^T\hat{\Phi}^j)^{-1}Z^T e_m^j,$$

where $\hat{\Phi}^j = \Psi(C_{fb}+C_{ff}^j)^{-1}y_m^j \in \mathbb{R}^{N\times n_\theta}$ and $Z = [\psi_1 r, \psi_2 r, \ldots, \psi_{n_\theta} r] \in \mathbb{R}^{N\times n_\theta}$.

The following result enables recursive estimation of θ.

Theorem 4. *For $C_{ff}^j, C_{ff}^{\Delta} \in \mathcal{C}$ with identical basis functions Ψ, C_{ff}^{j+1} in (1) is given by*

$$C_{ff}^{j+1} = \sum_{k=1}^{n_\theta} \psi_i \theta_i^{j+1}$$

where $\theta_i^{j+1} = \theta_i^j + \theta_i^{\Delta}$.

Proof. Follows by observing that C_{ff}^j and C_{ff}^{Δ} are linear in θ^j and θ^{Δ}, respectively. The details are omitted for brevity and will be provided elsewhere. □

Thm. 4 shows that the recursion of θ^{j+1} with respect to θ^j is solely based on measurements from the j^{th} task.

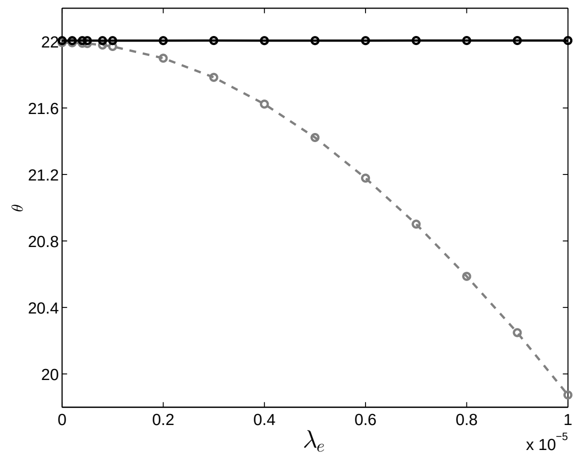

Fig. 3. The sample mean $\bar{\theta}^{LS}$ (dashed grey) and $\bar{\theta}^{IV}$ (black) resulting from Monte Carlo simulations as a function of standard deviation λ_ϵ show that $\bar{\theta}^{LS}$ is biased for $\lambda_e > 0$, while $\bar{\theta}^{IV}$ is unbiased $\forall \lambda_e$.

Combining Prop. 1 and Thm. 4 leads to the following procedure to update C_{ff}^j based on the j^{th} iteration, which constitutes the main contribution of this paper.

Procedure 1. *Estimation of $\hat{\theta}_N^{\Delta}$ in j^{th} iteration*

1) *Measure e_m^j and y_m^j for $t = 1, ..., N$.*
2) *construct $\hat{\Phi} = \Psi(q)(C_{fb}(q)+C_{ff}^j(q))^{-1}y^j$*
3) *construct instruments* $Z = [\psi_1 r, \quad \psi_2 r, \quad ..., \psi_{n_\theta} r]$
4) *solve $\hat{\theta}_N^{\Delta} = (Z^T\hat{\Phi})^{-1}Z^T e_m^j$.*
5) *Construct $C_{ff}^{j+1} = \Psi(q)(\hat{\theta}_N^j + \hat{\theta}_N^{\Delta})$*

Remark 4. *Similar to Remark 2, preview-based stable inversion [16] is used to compute $(C_{fb}(q)+C_{ff}^j(q))^{-1}y$.*

Proc. 1 provides a systematic procedure to improve the performance of the system in Fig. 1 by exploiting recursive estimates of θ to update $C_{ff}(q,\theta)$. In the next section, the theoretical results derived in this paper are illustrated in an simulation example.

VII. SIMULATION EXAMPLE

In this section, a simulation example is provided to confirm the claims posed in this paper with respect to the influence of v on $\hat{\theta}_N^{LS}$ and $\hat{\theta}_N^{IV}$, given by respectively (12) and (15). To this purpose, $C_{ff}(z,\theta) \in \mathcal{C}$ is determined by minimizing $V(C_{ff})$ under the constraint that $C_{ff}(z,\theta)|_{z=1} = 0$, i.e., the static gain of $C_{ff}(z,\theta)$ is equal to zero. Monte Carlo simulations are performed for numerical illustration. Define $\bar{\theta}^{LS,*}$ and $\bar{\theta}^{IV,*}$ as the asymptotic sample mean for $\lambda_\epsilon = 0$. Two cases are analyzed: i) $\bar{\theta}^{LS}$ and $\bar{\theta}^{IV}$ as a function of λ_e and ii) multiple tasks of $\bar{\theta}^{LS}$ and $\bar{\theta}^{IV}$ for a fixed λ_e.

Consider the plant P given by

$$P(z) = \frac{5.682\times 10^{-9}z + 5.682\times 10^{-9}}{z^2-2z+1},$$

which corresponds to the rigid-body dynamics of a motion system. This type of systems is researched in high-precision

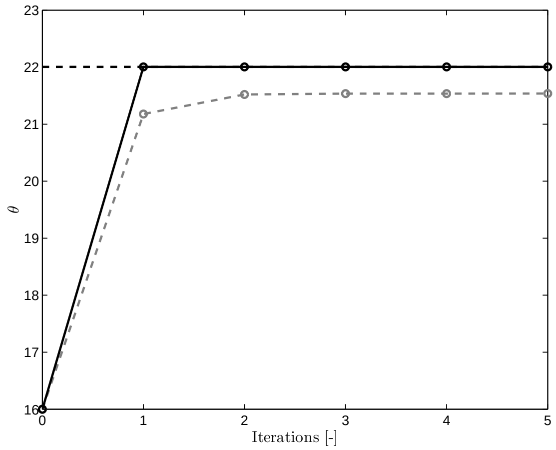

Fig. 4. $\bar{\theta}^{LS}$ (dashed grey) and IV-estimate $\bar{\theta}^{IV}$ (black) as a function of iteration for $\lambda_e = 6 \times 10^{-6}$ illustrate that iterating does not eliminate the bias of $\bar{\theta}^{LS}$, while $\bar{\theta}^{IV}$ is one-shot correct.

motion control, see, e.g., [17]. Furthermore, the feedback controller C_{fb} and noise filter H are given by

$$C_{fb}(z) = \frac{7.535 \times 10^4 z^2 - 1.488 \times 10^5 z + 7.348 \times 10^4}{z^3 - 2.736z^2 + 2.49z - 0.7537},$$
$$H(z) = \frac{0.505z^2 - 1.01z + 0.505}{z^2 - 0.7478z + 0.2722}.$$

The closed-loop system is excited by an 3^{th}-order point-to-point motion setpoint, see, e.g., [14]. Finally, the feedforward controller is parametrized as $C_{ff}(z, \theta) = \psi(z)\theta$, where $\psi(z) = \frac{z^2 - 2z + 1}{T_s^2 z^2}$, with sampling time $T_s = 5 \times 10^{-4}$ [s]. Furthermore, the number of samples and realizations are given by $N = 6000$ and $m = 100$, respectively.

First, $\bar{\theta}^{LS}$ and $\bar{\theta}^{IV}$ are analyzed as a function of λ_ϵ. The results depicted in Fig. 3 show that $\bar{\theta}^{LS} \neq \bar{\theta}^{LS,*}$ $\forall \lambda_e > 0$. This confirms the claims posed in Thm. 2, i.e., there exist a closed-loop identification problem if $V_2(\theta)$ is minimized. On the other hand, $\bar{\theta}^{IV} = \bar{\theta}^{IV,*}$, independent of λ_ϵ. This firmly confirms Thm. 3, showing that the IV method based on minimization of $V_z(\theta)$ eliminates the contribution of λ_e.

Second, multiple iterative tasks are considered for a fixed standard deviation $\lambda_e = 6 \times 10^{-6}$. For each iteration, $\hat{\theta}_N^{IV}$ is determined by means of Proc. 1. A similar procedure is used to determine $\hat{\theta}_N^{LS}$. Finally, $\bar{\theta}^{IV}$ and $\bar{\theta}^{LS}$ are determined for each iterative task. The results depicted in Fig. 4 show that only a single iteration is required to obtain $\bar{\theta}^{IV} = \bar{\theta}^{IV,*}$. This confirms the claims in Sect. VI. To the contrary, $\bar{\theta}_{LS} \neq \bar{\theta}^{LS,*}$ for all iterations. This shows that iterating does not resolve the bias of $\bar{\theta}_{LS}$ with respect to $\bar{\theta}^{LS,*}$.

VIII. CONCLUSIONS

In this paper, a novel approach for iterative feedforward control is presented that significantly enhances existing feedforward control algorithms. The presented approach: i) requires measured data from only a single task to update the feedforward controller and ii) attains optimal performance for feedforward control in the presence of noise. Simulation results confirm that the proposed iterative feeforward control approach is superior compared to pre-existing results.

The proposed approach can be straightforwardly extended to other optimization problems in a closed-loop configuration. Extensions to recursive parameter estimation and multivariable systems are beyond the scope of the present paper.

IX. ACKNOWLEDGEMENTS

The authors acknowledge Okko Bosgra, Dennis Bruijnen and Maarten Steinbuch for valuable discussions and their contribution to this work.

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