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Design Techniques for Multivariable ILC: Application to an Industrial Flatbed Printer

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Abstract: Iterative Learning Control (ILC) can significantly improve performance of systems that perform repeating tasks. Although in practice many systems are multivariable, frequency-domain ILC design procedures often involve multi-loop single-input single-output (SISO) filters which do not explicitly address interaction in dynamics. The aim of this paper is to i) analyze multi-loop SISO ILC designs, ii) point out the importance of multivariable ILC design, and iii) develop the required multivariable design algorithms. Benefits of the proposed approaches over multi-loop SISO ILC are demonstrated on an industrial printer model example.

Keywords: Iterative learning control, multivariable control, zero phase-error tracking control, non-minimum phase systems, motion control, $\mathcal{H}_\infty$ control, preview control

1. INTRODUCTION

Iterative Learning Control (ILC) (Bristow et al. (2006)) is widely used in control systems, since it can significantly improve the performance of systems that perform repeating tasks. Many successful applications have been reported, including additive manufacturing (Barton et al. (2011); Hoelzle et al. (2011)), industrial robots, printing systems (Bolder et al. (2015)), and wafer stages (Mishra et al. (2008)).

Important design frameworks in ILC include frequency-domain design, and time-domain norm-optimal design. The time-domain norm-optimal framework often uses advanced synthesis tools from optimal control, and requires detailed uncertainty models to provide robustness, see, e.g., Ahn et al. (2007); Van de Wijdeven et al. (2009). In contrast, frequency-domain ILC design can be based on nonparametric frequency response function (FRF) measurements, and performance and robustness requirements can be enforced by means of standard frequency-domain loop-shaping methods, see, e.g., Bristow et al. (2006, 2010); Moore (1993); Boeren et al. (2016). These are important advantages for industrial motion control applications. Hence, the present paper focuses on frequency-domain ILC design techniques.

Although in practice many systems are multivariable, i.e., multi-input multi-output (MIMO), design techniques for ILC often consider multi-loop single-input single-output (SISO) controllers, see, e.g., Bristow et al. (2006); Moore (1993); Wallén et al. (2008). In these multi-loop SISO design techniques, interactions in MIMO systems are not explicitly taken into account, which potentially can lead to non-convergent algorithms. To deal with these ignored dynamics, performance of the ILC may have to be sacrificed due to the more severe demand on robustness. This is a well known trade-off between performance and robustness.

Noncollocated sensors and actuators, and fast sample rates with plants having high relative degree can lead to non-minimum phase (NMP) dynamics which complicate the ILC design. For NMP systems, inputs obtained through standard inversion techniques typically yield unbounded outputs. Solutions to compute bounded outputs include stable inversion (Blanken et al. (2016a); Boeren et al. (2015); Bolder et al. (2015)), and heuristic stable designs such as the widely used zero phase-error tracking control (ZPETC) of Tomizuka (1987), the related zero magnitude-error tracking control (ZMETC), see, e.g., Butterworth et al. (2012), and perfect tracking control, see, e.g, Fujimoto et al. (2001). Recently, in Blanken et al. (2016b) an intuitive procedure is proposed for computing full MIMO controllers which provide phase cancellation for NMP zeros. Though, this procedure requires performing pole-zero cancellations, which can be numerically troublesome.

In this paper, two approaches for multivariable ILC design are proposed. First, an alternative implementation is presented with improved numerical properties compared to the heuristic design of Blanken et al. (2016b). This procedure is applicable to MIMO systems, and recovers the traditional ZPETC for SISO systems. Second, a procedure is presented for $\mathcal{H}_\infty$-optimal ILC synthesis with finite preview. For zero preview, this approach recovers the results of De Roover and Bosgra (2000). This approach potentially outperforms heuristic designs in terms of achievable performance, and numerical properties.

The aim of this paper is to point out the importance of multivariable ILC design techniques related to the performance-robustness trade-off, and develop the required algorithms. By explicitly incorporating interactions in multivariable ILC design, performance can potentially be improved, see, e.g., De Roover and Bosgra (2000). The contributions of this paper are fourfold. First, a design-oriented analysis is presented of multi-loop SISO ILC, related to robustness for ignored dynamics. Second, an algorithm is proposed to design a multivariable learning filter for NMP systems, which cancels the phase shifts induced by unstable zeros. Third, an $\mathcal{H}_\infty$-based approach is presented for the optimal synthesis of ILC systems, including finite preview. Fourth, the benefits are demonstrated of explicitly designing the ILC for interaction on an industrial printer model example.

The outline of this paper is as follows. In Section 2, the MIMO frequency-domain ILC design problem is formulated. In Section 3, a design-oriented analysis of multi-loop SISO ILC is presented. In Section 4, a heuristic MIMO ILC design is presented, and in Section 5 an $\mathcal{H}_\infty$-based approach is presented for the optimal synthesis of ILC systems with finite preview. In Section 6, the industrial flatbed printer used for simulations is introduced, for which MIMO ILCs are designed in Section 7. In Section 8, the benefits of MIMO ILC are demonstrated by use of simulations. Finally, conclusions are provided in Section 9.

Notation: $\mathbb{R}[z]$ denotes the polynomial ring in indeterminate z with coefficients in $\mathbb{R}$. $\mathcal{R}(z)$ denotes the field of real rational functions. The space consisting of all square summable sequences is denoted ℓ_2. Given $f(z), g(z) \in \mathbb{R}[z]$, $g(z)$ divides $f(z)$ if there exists a $h(z) \in \mathbb{R}[z]$ such that $f(z) = g(z)h(z)$. A polynomial is called monic if it has leading coefficient 1. A polynomial matrix $U(z) \in \mathbb{R}^{n\times n}[z]$ is called unimodular if and only if $U^{-1}(z) \in \mathbb{R}^{n\times n}[z]$. A_d denotes the diagonal matrix containing the diagonal elements of A. Furthermore, $\mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ is the diagonal matrix with diagonal elements $a_1, a_2, \ldots, a_n$. Throughout, all systems are assumed to be discrete-time, multi-input multi-output (MIMO), and linear time-invariant. The complex indeterminate z is omitted when this does not lead to any confusion.

2. PROBLEM FORMULATION

In this section, the multivariable ILC design problem is formulated, and design challenges are indicated motivating the analysis and development of multivariable ILC design techniques in Sections 3 and 4, respectively.

Consider the control configuration depicted in Figure 1, consisting of the true system $P(z) \in \mathcal{R}^{p\times q}(z)$ and a stabilizing feedback controller $C(z) \in \mathcal{R}^{q\times p}(z)$. The system is repeatedly excited by a reference signal r. Each repetition of r is called a task, denoted by subscript j. Furthermore, f_j denotes the feedforward, u_j the controller output, y_j the output, and e_j the error in task j, given by

$$e_j = Sr - SPf_j,$$

with sensitivity $S = (I + PC)^{-1} \in \mathcal{R}^{p\times p}(z)$. The objective of ILC is to minimize e_j in terms of an appropriate norm. To this purpose, e_j is measured in task j and used to construct f^{j+1} in task $j + 1$. Typically, the following general ILC algorithm is invoked:

$$f_{j+1} = Q(f_j + Le_j), \tag{1}$$

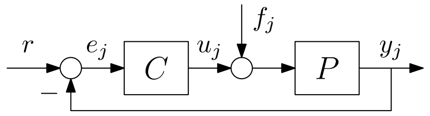


Fig. 1. ILC control configuration.

with learning filter $L \in \mathbb{R}^{q\times p}$ and robustness filter $Q \in \mathbb{R}^{q\times q}$. A condition for monotonic convergence of f_j is given in the following theorem.

Theorem 1. Consider the control configuration in Figure 1, and suppose $u_j, e_j \in \ell_2$, then the ILC algorithm (1) converges monotonically to a fixed point f^* if

$$\|Q(I - LSP\|_\infty < 1 \tag{2}$$

where $\|H\|_\infty = \sup_{\omega\in[0,\pi]} \bar{\sigma}(H(e^{j\omega}))$ is the $\mathcal{L}_\infty$-norm, and $\bar{\sigma}$ denotes the maximum singular value.

Proof. A proof of Theorem 1 is omitted, since it follows along similar lines as (Moore, 1993, Theorem 3.1) for the $\mathcal{H}_\infty$-norm, and can be appropriately extended to the $\mathcal{L}_\infty$-norm to account for noncausal L, Q using (Chen and Gu, 2000, Theorem 2.1.10). □

If (2) is satisfied, the fixed point f^* is given by

$$f^* = (I - Q(I - LSP))^{-1} QLSr,$$

and the resulting fixed point of the error is given by

$$e^* = \Big(I - SP\,(I - Q(I - LSP))^{-1}\, QL\Big)\, Sr. \tag{3}$$

For (2) to be satisfied and (3) minimal, it can be seen that L should be chosen equal to $(SP)^{-1}$ with $Q = I$. However, NMP dynamics of SP can complicate the control design.

Although convergence analysis results as Theorem 1 have received considerable attention in literature, practical procedures for the design of multivariable filters L and Q are difficult to find. Often, L and Q are designed as multi-loop SISO filters, see Bristow et al. (2006); Moore (1993). This design choice for ILC is analyzed in the next section. This analysis will motivate to develop design procedures for *full* multivariable L-filters in Sections 4 and 5.

3. ANALYSIS & DESIGN OF MULTI-LOOP SISO ILC

In this section the design of multi-loop SISO, or diagonal, ILC is addressed. Though this makes the design procedure relatively simple, interaction in the closed-loop system is ignored. Hence, the robustness filter Q must not only be designed for model uncertainty, but also for interaction.

A multi-loop SISO learning filter is of the form

$$L = \mathrm{diag}\{L_1, L_2, \ldots, L_p\}, \tag{4}$$

with $L_i \in \mathcal{R}(z)$. Each element L_i can, e.g., be designed as the zero-phase error tracking controller for the corresponding element of $(\widehat{SP})_\mathrm{d}$, as in Tomizuka (1987). Given this design approach for L, the design of robustness filter Q is investigated in the following sections.

3.1 Multi-Loop SISO Design for Q Ignoring Interaction

In this section, the design of a multi-loop SISO robustness filter Q is analyzed, with attention to model uncertainty and interaction in the dynamics. Here, model uncertainty refers to $\Delta = (SP)_\mathrm{d} - (\widehat{SP})_\mathrm{d}$, i.e., the model error in the elements of $(\widehat{SP})_{\mathrm{d}}$ relevant for the design of L. Interaction refers to $(SP)_{\mathrm{nd}} = SP - (SP)_{\mathrm{d}}$, i.e., the off-diagonal elements of SP that are ignored in the design of L.

A multi-loop SISO robustness filter is of the form

$$Q = \mathrm{diag}\{Q_1, Q_2, \ldots, Q_q\},$$

with $Q_i \in \mathcal{R}(z)$. To enforce robustness with respect to model uncertainty Δ, each Q_i must be designed to satisfy

$$\left|Q_i(e^{j\omega})(1 - L_i(e^{j\omega})(SP)_{ii}(e^{j\omega}))\right| < 1, \quad \forall i, \omega \in [0, 2\pi],$$

which is equivalent to

$$\bar{\sigma}\left(Q(e^{j\omega})(I - L(e^{j\omega})(SP)_{\mathrm{d}}(e^{j\omega}))\right) < 1, \forall \omega \in [0, 2\pi], \quad (5)$$

where $\bar{\sigma}(\cdot)$ denotes the maximum singular value of $(\cdot)$. For the SISO case, condition (5) is sufficient for monotonic convergence of the ILC system, since $(SP)_{\mathrm{d}} = SP$. However, in the MIMO case, interaction in the closed-loop system is ignored. In the next section, this is analyzed and a design for Q is proposed which *robustifies* the system.

3.2 Interaction Accounted for Through Robustness

It follows from Theorem 1 that, for MIMO systems, a sufficient condition for monotonic convergence is given by

$$\begin{aligned}
&\bar{\sigma}\left[Q(e^{j\omega})(I - L(e^{j\omega})SP(e^{j\omega}))\right] \\
&= \bar{\sigma}\Big[Q(e^{j\omega})(I - L(e^{j\omega})(SP)_{\mathrm{d}}(e^{j\omega})) \\
&\quad - Q(e^{j\omega})L(e^{j\omega})(SP)_{\mathrm{nd}}(e^{j\omega})\Big] < 1, \quad \forall \omega \in [0, 2\pi]. \quad (6)
\end{aligned}$$

Note that the difference between (5) and (6) is the term $QL(SP)_{\mathrm{nd}}$ in the argument of $\bar{\sigma}(\cdot)$. This is exactly the influence of interaction on the closed-loop system. If this interaction term is sufficiently large, the multi-loop ILC system designed according to (5) can become unstable.

To *robustify* the closed-loop system, Q can be designed to account for ignored interaction. For instance, select $Q = Q_{\mathrm{d}}I$ with $Q_{\mathrm{d}} \in \mathcal{R}(z)$. Then, the MIMO system is monotonically convergent if Q_{d} is designed such that

$$\begin{aligned}
&\bar{\sigma}\left(Q(e^{j\omega})(I - L(e^{j\omega})S(e^{j\omega})P(e^{j\omega}))\right) \\
&\le \bar{\sigma}(Q(e^{j\omega})\bar{\sigma}\left(I - L(e^{j\omega})S(e^{j\omega})P(e^{j\omega})\right) \\
&\quad = \left|Q_{\mathrm{d}}(e^{j\omega})\right| \bar{\sigma}\left(I - L(e^{j\omega})S(e^{j\omega})P(e^{j\omega})\right) \\
&< 1 \quad \forall \omega \in [0, 2\pi]. \quad (7)
\end{aligned}$$

Though this design procedure renders the multivariable system stable, the use of Q hampers the achievable performance. That is, the robustness required to compensate for ignored interaction comes at the cost of performance.

Remark 2. Note that the feedback controller C forms a degree-of-freedom in the closed-loop transfer matrix SP. Interestingly, C can thus be exploited to attenuate interaction in the closed-loop system, and to increase the achievable performance of a multi-loop SISO ILC system.

3.3 Design for Interaction

Although the presented multi-loop SISO ILC design procedure is relatively simple and makes use of well known design tools, the performance of the system may be compromised due to Q-filtering for neglected interaction. An approach to overcome this is to design a *full* multivariable L-filter that explicitly takes interaction into account.

Systems with NMP dynamics complicate the ILC design, since $L = (SP)^{-1}$ is unstable. To deal with unstable multivariable $(SP)^{-1}$, several approaches have been developed. In Boeren et al. (2015); Blanken et al. (2016a), an approach is proposed which enables implementation of unstable L-filters through stable inversion techniques. In Van Zundert et al. (2016), a practical design approach is presented which, when applied for an infinite iteration length, can be used to compute bounded feedforwards f_{j+1} for frequency-domain ILC schemes.

Alternatively to these approaches, in the next section a novel approach is presented for the design of a stable L-filter which provides phase cancellation for unstable transmission zeros of SP. This approach shows strong similarities with the ZPETC of Tomizuka (1987) for SISO systems, yet is developed for multivariable systems.

4. HEURISTIC DESIGN OF A STABLE MULTIVARIABLE LEARNING FILTER: ZPETC

In this section, a heuristic approach is presented to compute a stable learning filter, denoted by $L \in \mathcal{R}^{q \times p}$, for the multivariable system $SP \in \mathcal{R}^{p \times q}$. The stable learning filter provides phase cancellation for unstable zeros of SP, and simplifies in the SISO case to the ZPETC of Tomizuka (1987).

The presented procedure is an alternative implementation of Blanken et al. (2016b). The approach presented there exploits the Smith-McMillan form of rational transfer matrices, which is a valuable conceptual tool. However, this requires performing pole-zero cancellations, which is well-known to be numerically troublesome. Here, a procedure is presented, for which no pole-zero cancellations are required. To this purpose, an irreducible matrix fraction description (MFD) of G is constructed in Section 4.1. Based on the zeros of G, a multivariable ZPETC for the numerator matrix is developed in Sections 4.2 and 4.3. Finally, the stable learning filter for G is constructed in Section 4.4.

4.1 Step 1: Construct Irreducible Right MFD

In this section, an irreducible MFD of $G(z)$ is constructed. For the application to ILC, specifically a right MFD of G, which represents SP, is required. The filter L multiplies SP from the left in (2). To cancel the phase shifts induced by unstable zeros, these zeros must thus be accessed from the left, motivating the use of a *right* MFD.

Consider a state-space representation

$$G \overset{s}{=} \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right],$$

where $D = 0$ is assumed without loss of generality. Convert the left MFD $(zI - A)^{-1}B = D_L^{-1}(z)N_L(z)$ to an irreducible right MFD $E_R(z)D_R^{-1}(z)$. This conversion can be written, see, e.g., (Kailath, 1980, Lemma 6.3-9), as

$$[-N_L(z) \;\; D_L(z)] \begin{bmatrix} D_R(z) \\ E_R(z) \end{bmatrix} = 0.$$

In other words, N_R and D_R belong to the right null space of the polynomial matrix $[-N_L \;\; D_L]$. Of all candidate matrices $[D_R\ E_R]^\top$, there is a minimal polynomial basis, i.e., that has the smallest possible row degrees. This right MFD $E_R D_R^{-1}$ is irreducible, see, e.g., Kailath (1980). Numerically stable algorithms exist to compute minimal polynomial bases, see, e.g., Basilio and Moreira (2004); Zúñiga Anaya and Henrion (2009). This result allows to write G as an irreducible right MFD, given by

$$G = N_R D_R^{-1}, \quad \text{with } N_R = CE_R. \tag{8}$$

The following definitions are key for subsequent results.

Definition 3. A complex number z_0 is called a pole of $G(z)$ if $\det D(z_0^{-1}) = 0$, where $D(z)$ is the denominator of any irreducible MFD of $G(z)$.

Definition 4. A complex number z_0 is called a zero of $G(z)$ if $\text{rank}N(z_0^{-1}) < r$, where r is the normal rank of $G(z)$, and $N(z)$ is the numerator of any irreducible MFD of $G(z)$.

4.2 Step 2: Compute Smith Form of N_R and Factorize Multivariable Zeros

It follows from Definition 4 that N_R in (8) contains the zeros of G. To directly access these zeros, N_R can be reduced to its Smith form $\mathcal{S}(z)$, see, e.g., Kailath (1980).

Lemma 5. (Smith form). Let $N(z) \in \mathbb{R}^{p\times q}[z]$ be of normal rank r, Then, there exist unimodular matrices $U(z) \in \mathbb{R}^{p\times p}[z]$, $V(z) \in \mathbb{R}^{q\times q}[z]$ such that

$$\begin{aligned} U(z)N(z)V(z) &= \mathcal{S}(z) \\ &= \begin{bmatrix} \varepsilon_1'(z) & 0 & \cdots & 0 & 0 \\ 0 & \varepsilon_2'(z) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \varepsilon_r'(z) & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \end{aligned}$$

where each $\varepsilon_i'(z) \in \mathbb{R}[z]$ is monic and $\varepsilon_i'(z)$ divides $\varepsilon_{i+1}'(z)$ for $i = 1, \dots, r-1$. The $\varepsilon_i'(z)$ are called the invariant factors of $N(z)$, which are uniquely defined by

$$\varepsilon_i'(z) = \frac{\Delta_i(z)}{\Delta_{i-1}(z)}, \quad i = 1, \dots, r,$$

where $\Delta_0 = 1$ and Δ_i is defined as the greatest common divisor of all $i \times i$ minors of $N(z)$.

By reducing N_R to its (unique) Smith form

$$\mathcal{S} = UN_RV, \tag{9}$$

see Lemma 5, the stable and unstable zeros of G can be factorized. That is, each nonzero element on the diagonal of $\mathcal{S}$ is written as

$$\mathcal{S}_i(z) = z^{-d_i}\varepsilon_{i,s}(z)\varepsilon_{i,u}(z), \quad i = 1, \dots, r$$

where the integer d_i represents a pure delay or time advance, $\varepsilon_{i,s}$ contains the remaining zeros of ε_i strictly inside the open unit disk, and $\varepsilon_{i,u}$ contains the zeros of ε_i strictly outside the open unit disk.

4.3 Step 3: Compute ZPETC for Smith Form of N_R

In this section, a ZPETC is proposed for $\mathcal{S}$. Considering each element on the diagonal of $\mathcal{S}$ as an independent single-input single-output subsystem, we can apply the ZPETC algorithm for SISO systems, developed by Tomizuka (1987), on each $\mathcal{S}_i$, $i = 1, \dots, r$. For element $\mathcal{S}_i$, the ZPETC $L_{\mathcal{S},i}$ is given by

$$L_{\mathcal{S},i}(z) = z^{d_i}\frac{\varepsilon_{i,u}(z^{-1})}{\beta_i\varepsilon_{i,s}(z)}, \qquad i = 1, \dots, r.$$

This leads to

$$L_{\mathcal{S},i}(z)\mathcal{S}_i(z) = \frac{\varepsilon_{i,u}(z^{-1})\varepsilon_{i,u}(z)}{\beta_i}, \qquad i = 1, \dots, r,$$

which has zero phase. Parameter β_i can be chosen such that $|L_{\mathcal{S},i}(z)\mathcal{S}_i(z)| = 1$ at some frequency. E.g., choose $\beta_i = \varepsilon_{i,u}^2(1)$ for $L_{\mathcal{S},i}(e^{j\omega})\mathcal{S}_i(e^{j\omega}) = 1$ at $\omega = 0$.

4.4 Step 4: Construct Stable Learning Filter

In the previous step, $L_{\mathcal{S}}$ is designed based on $\mathcal{S}$ in (9). Next, filter L is constructed which cancels the phase shifts induced by unstable zeros of G. To achieve this, select

$$L = D_RVL_{\mathcal{S}}U. \tag{10}$$

Then, it holds

$$\begin{aligned} LG &= L\left(U^{-1}\mathcal{S}V^{-1}D_R^{-1}\right) \\ &= D_RVL_{\mathcal{S}}\mathcal{S}V^{-1}D_R^{-1}. \end{aligned}$$

Three observations are made. First, L is stable since $L_{\mathcal{M}}$ is stable and U, V are polynomial matrices. Second, in case G has no unstable zeros, $LG = I$ for all frequencies, and perfect tracking is achieved. Third, in case G has unstable zeros, it holds $L_{\mathcal{M}}\mathcal{M} \neq I$ for some frequencies. Although $L_{\mathcal{M}}\mathcal{M}$ has zero phase, there is no guarantee that LG has zero phase due to multiplications with transfer matrix V.

Remark 6. The obtained learning filter $L(z)$ can be noncausal. For the application to ILC this does not pose a problem, since typically the computation of (1) is performed off-line and e_j is known beforehand.

4.5 Equivalence of Implementations

The key benefit of the implementation presented in this section compared to that in Blanken et al. (2016b) is that no pole-zero cancellations are required. Though, from a theoretical viewpoint, both implementations are equivalent. This follows from the fact that the Smith-McMillan form of a rational transfer matrix is closely related to an MFD. In fact, the Smith-McMillan form of a transfer matrix is equivalent to an irreducible MFD, e.g., (8).

In the next section, the theoretical results derived in this section are illustrated with a numerical example.

4.6 Numerical Example

In this section, a numerical example is provided to illustrate the procedure described in Section 4 to compute a stable learning filter for a multivariable system.

Example 7. Consider the rational proper transfer matrix

$$G(z) = \begin{bmatrix} \dfrac{k_1(z-0.73)}{z(z-0.97)} & \dfrac{k_2(z-0.97)}{z(z-0.73)} \\ \dfrac{k_2(z-0.97)}{z(z-0.73)} & \dfrac{k_1(z-0.73)}{z(z-0.97)} \end{bmatrix}, \tag{11}$$

with k_1, k_2 such that $G_{11}(e^{i0}) = 1$ and $G_{12}(e^{i\pi}) = 1$. Analysis of the MIMO transfer matrix (11) reveals

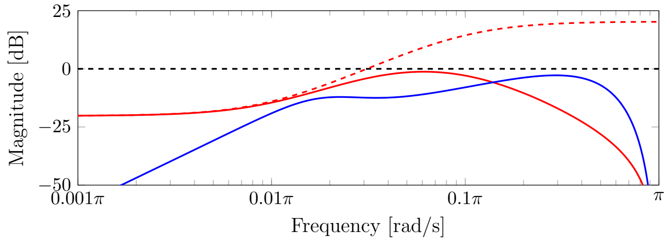


Fig. 2. $\bar{\sigma}(Q(I-LG))$ for MIMO approach (—), and multi-loop SISO approach with $Q = Q_\mathrm{d} I$ (—) and $Q = I$ (– –).

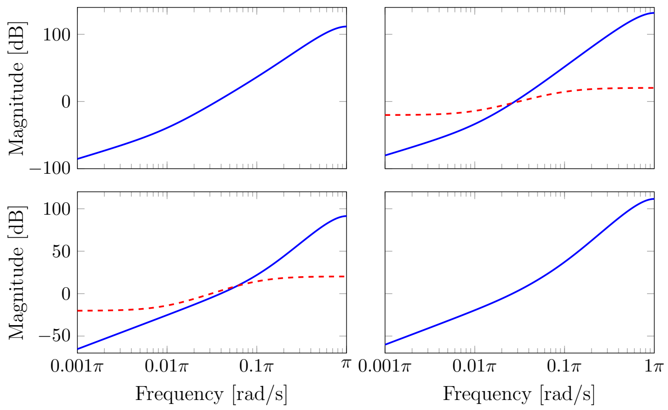


Fig. 3. Further interpretation of Figure 2: the Bode diagram of $I-LG$ indicates that the MIMO L-filter (—) improves tracking, i.e. $LG \approx I$, up to the unstable zero at $\omega = 0.031\pi$ rad/s by accounting for interaction, whereas the multi-loop SISO approach (– –) ignores interaction and the unstable zero of G.

that G has an unstable transmission zero at $z = 1.102$ and frequency $\omega = 0.031\pi$ rad/s. This is an essential observation, since the zero is not directly observed in (11).

A MIMO L-filter (10) and a multi-loop SISO L-filter (4) are designed. If the Q-filter for the multi-loop SISO approach is designed while ignoring all interaction, see (5), $Q = I$ suffices. However, Figure 2 demonstrates that the multivariable system for this case is in fact nonconvergent. To *robustify* the system against interaction, see (7), a filter $Q = Q_\mathrm{d} I$ is required with Q_d a zero-phase low-pass 2^{nd} order Butterworth filter with a cut-off frequency of 0.2π rad/s. For the MIMO L-filter, $Q = Q_\mathrm{d}^2 I$ is used to render the multivariable system monotonically convergent.

Figure 3 demonstrates that the MIMO L-filter improves tracking up to the frequency of the unstable zero by accounting for interaction, whereas the multi-loop SISO approach only compensates for the diagonal elements of G, hence ignoring all interaction and the unstable transmission zero (which is not observed in (11)).

4.7 Concluding Remarks

In this section, a heuristic algorithm is described to design a multivariable stable learning filter L, which cancels the phase shifts induced by unstable zeros. For SISO systems, the approach recovers the ZPETC of Tomizuka (1987). The approach provides an alternative implementation for the procedure presented in Blanken et al. (2016b), which requires numerically troublesome pole-zero cancellations, yet still requires the computation of a Smith form. Typical numerical methods to compute Smith forms make use of elementary row and column operations, see, e.g., Kailath (1980). A major caveat of these methods is numerical stability, lost because of pivoting with respect to power monomials of z, see Gantmacher (1959). Indeed, algorithms that compute the Smith form of polynomial matrices *with* multipliers U, V, without relying on elementary row and column operations are developed, and can be found in, e.g., Kaltofen et al. (1990); Villard (1995); Wilkening and Yu (2011).

The proposed heuristic L-filter design, presented in this section, provides a specific stable approximation of $(SP)^{-1}$. Yet, it is unsure whether this design is an *optimal* stable approximation of $(SP)^{-1}$. In the next section, a systematic method is proposed to synthesize an optimal stable approximation of $(SP)^{-1}$.

5. ALTERNATIVE APPROACH FOR ILC DESIGN: $\mathcal{H}_\infty$-SYNTHESIS WITH FINITE PREVIEW

In this section, a systematic procedure for the synthesis of iterative learning controllers is presented within the $\mathcal{H}_\infty$ mathematical framework, see, e.g, Zhou et al. (1996). As a key feature, learning filters with finite preview can be designed. This aspect is crucial for the potential performance of ILC schemes.

5.1 Preview in Learning Filters

Essential for the performance of ILC for strictly proper and non-minimum phase systems is the use of preview, since this directly enables non-causal feedforward signals in the physical time domain, see Bristow et al. (2006) for the relevant definition. The ZPETC approach in Section 4 uses a finite amount of preview to provide phase cancellation for unstable zeros of SP. Though, this approach requires the numerically troublesome computation of a Smith form, see Section 4.7. As an alternative to this approach, recently an LQ-optimal approach is developed Van Zundert et al. (2016) that enables infinite preview.

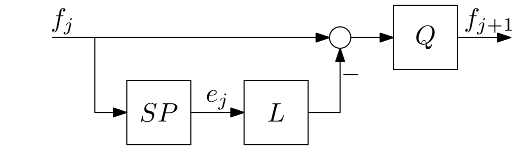
The aim of this section is to briefly highlight the potential of a finite preview optimal feedforward controller in the line of Van Zundert et al. (2016), yet in an LTI framework.

5.2 Approach: $\mathcal{H}_\infty$-Synthesis of L with Finite Preview

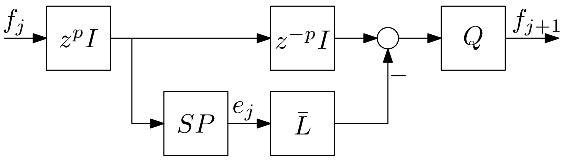
In this subsection, an $\mathcal{H}_\infty$-optimal approach is presented for the synthesis of filters L with finite preview. Consider the condition for monotonic convergence, see Theorem 1:

$$\|Q(I - LSP)\|_\infty < 1, \qquad (12)$$

where the $\mathcal{L}_\infty$ norm is used to allow for noncausal L, Q. It then follows that the resulting fixed point of the error e^*, see (3), is zero if and only if (12) holds and $Q = I$. In practice, the use of $Q = I$ is hampered by modeling errors and possible nonminumum-phase behavior in SP. In such cases, the use of a Q-filter *robustifies* the learning scheme, see also Section 3. Typically, the Q-filter is designed as a lowpass filter.



(a) Feedforward propagation.



(b) Feedforward propagation with *a priori* selected preview z^p.

Fig. 4. Feedforward propagation structure with finite preview for $\mathcal{H}_\infty$ optimal synthesis of L.

In view of these arguments, it seems natural to view Q as a measure for performance: the cut-off frequency of Q should be selected as high as possible, whilst (12) is satisfied. Here, the Q-filter is assumed to be already designed, and the focus is on the design of L.

The considered feedforward propagation in Theorem 1 is schematically depicted in Figure 4. The key point in the approach is that the input f_j is previewed with p samples. Then, the $\mathcal{H}_\infty$-synthesis problem is given by:

$$\bar{L}(z) = \arg \min_{\bar{L}(z)\in\mathcal{H}_\infty} \left\| Q(z^{-p}I - \bar{L}SP) \right\|_\infty .$$

Details on this optimal $\mathcal{H}_\infty$-synthesis problem and extensions will be published elsewhere, and are beyond the scope of the present paper. In fact, the proposed approach is an optimal replacement of the ZPETC filter, and is closely related to optimal preview control and fixed-lag smoothing, see, e.g., Hazell and Limebeer (2008); Kojima (2005); Marro and Zattoni (2005); Mirkin (2003); Moelja and Meinsma (2006).

Finally, the optimal learning filter $L \in \mathcal{RL}_\infty$ is obtained by applying preview p, as in the ZPETC case, to the obtained optimal controller $\bar{L} \in \mathcal{RH}_\infty$:

$$L = z^p \bar{L}.$$

Remark 8. In the specific case that p is selected as $p = 0$, the design problem reduces to a standard $\mathcal{H}_\infty$ optimization problem, and the results presented by De Roover and Bosgra (2000) are recovered.

In Sections 4 and 5, two different approaches are proposed for the design of stable multivariable ILC learning filters. In the next sections, the developed approaches are validated on an industrial motion system, and compared to the multi-loop SISO designs in Section 3.

6. SETUP: SYSTEM AND MODEL

In this section, the setup used for multivariable ILC design and simulations in Sections 7 and 8 is introduced. The system under consideration is an Océ Arizona 550 GT flatbed printer, see Figure 5. The system is controlled in four degrees of freedom (DOFs): the carriage translates in y (parallel the gantry) and z, the gantry translates in x and rotates in φ in the horizontal plane. In the remainder, only the translation in x and rotation in φ of the carriage

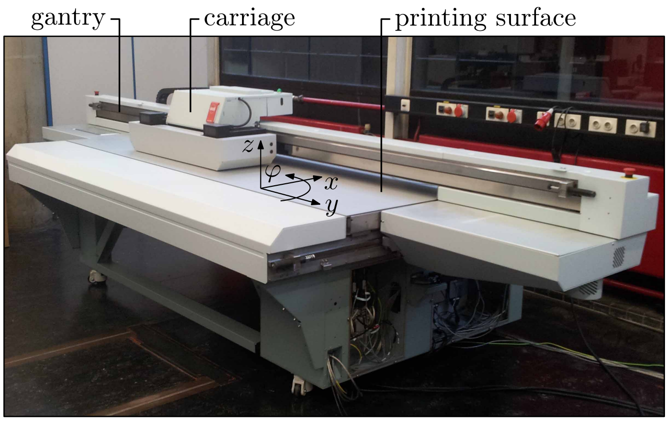


Fig. 5. Océ Arizona 550 GT at the CST Motion laboratory, Eindhoven University of Technology. The x and φ positions of the carriage are considered for control.

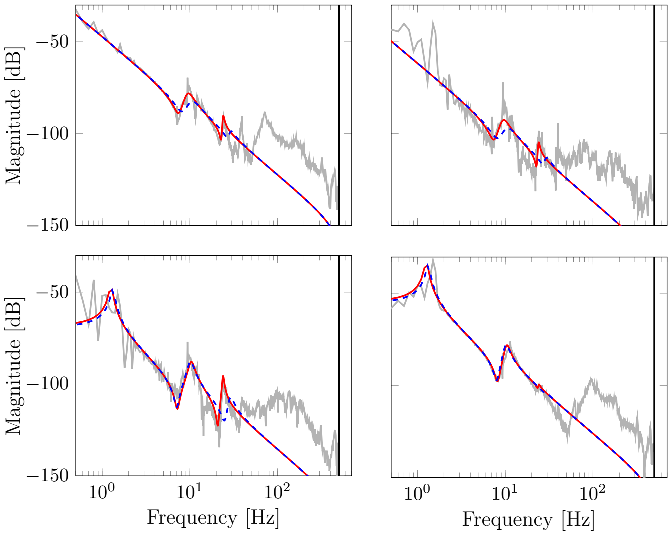


Fig. 6. Bode diagram of FRF (—), plant model P (—) and perturbed plant model $\widehat{P}$ (– –).

are considered. The inputs of the corresponding system are currents to the motors [A]; the outputs are the position of the carriage in [m] and [rad], respectively. A 12th order parametric model P, derived based on first principles, is constructed for use in simulations. A model $\widehat{P}$ with perturbed parameters is used for ILC design. The Bode diagram of P, $\widehat{P}$, and a frequency response function (FRF) measurement is depicted in Figure 6. The sampling time is $T_s = 0.001$ s. A stabilizing diagonal feedback controller is designed consisting of a lead-filter in each loop, yielding bandwidths of 6 Hz in x direction and 5.5 Hz in φ direction. The transfer matrix $\widehat{SP} = (I + \widehat{P}C)^{-1}\widehat{P}$ has two unstable zeros at $z = -1.44$ induced by delays in the closed loop.

7. DESIGN OF MULTIVARIABLE ILCS

In this section, multivariable ILCs are designed for the system described in Section 6, according to the procedures described in Section 4 and Section 5.

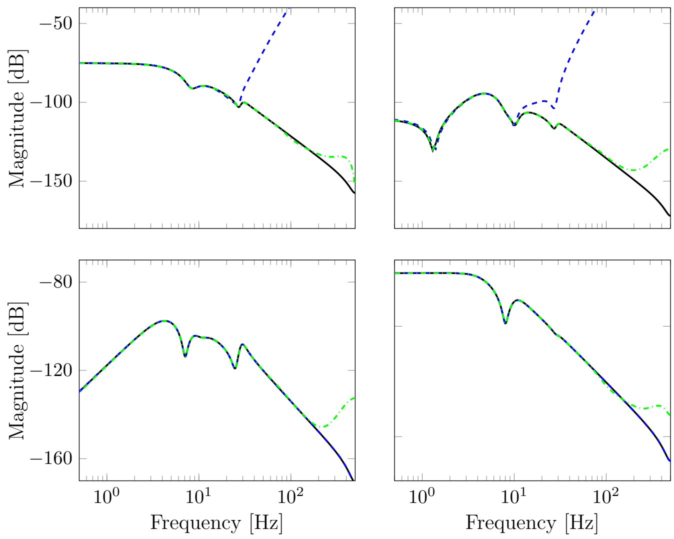


Fig. 7. Process sensitivity $\widehat{SP}$ (—), and inverse multivariable learning filters L_{ZPETC}^{-1} (– –) and $L_{\mathcal{H}_\infty}^{-1}$ (—).

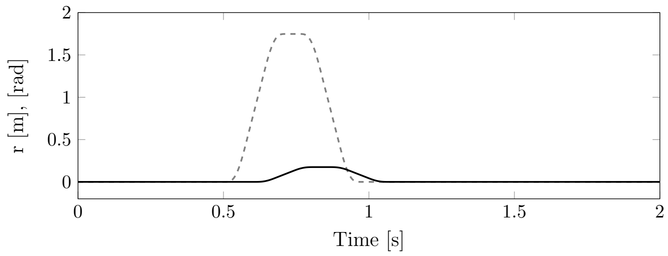


Fig. 8. Fourth order reference signals r^x (– –) and r^φ (—).

7.1 Heuristic Approach: ZPETC

A multivariable learning filter L_{ZPETC} is designed for the plant model $\widehat{P}$ depicted in Figure 6, using the heuristic approach in Section 4. Operations on polynomials are performed with the Polynomial Toolbox of PolyX, Ltd. (2000) in Matlab™. A Bode diagram of L_{ZPETC}^{-1} and model $\widehat{SP}$ is depicted in Figure 7. It is observed that L_{ZPETC}^{-1} accurately matches $\widehat{SP}$ up to 10 Hz. The deviation above this frequency is caused by numerical inaccuracies in U and V, which introduce gain and phase errors with respect to $\widehat{SP}$. These errors originate from the reduction of N_R to its Smith form in (9). Further work is required to obtain a numerically accurate Smith form, as in, e.g., Kaltofen et al. (1990); Villard (1995); Wilkening and Yu (2011).

Next, a robustness filter Q is designed. Here, Q is designed as a multi-loop SISO filter $Q = Q_\text{d} I$ with $Q_\text{d} \in \mathcal{R}(z)$, as in Section 3.2. Then, the multivariable ILC system is monotonically convergent if (7) is satisfied. The Q_d-filter is designed as a low-pass 4^th order Butterworth filter with a cut-off frequency of 23.5 Hz, which is applied using the Matlab™ function `filtfilt` to get zero-phase behaviour. The high order of the filter is required to provided robustness with respect to the inaccurate Smith form used in the computation of L_{ZPETC}.

7.2 $\mathcal{H}_\infty$-Optimal Approach with Finite Preview

Two ILC schemes are designed using the $\mathcal{H}_\infty$-optimal approach described in Section 5. For a fair comparison with the heuristically designed filter L_{ZPETC} in the previous subsection, a finite preview of $p = 3$ samples is used, which equals the preview of L_{ZPETC}.

First, an ILC scheme is designed for superior performance. That is, the cut-off frequency of the low-pass Butterworth filter filter Q_d in $Q = Q_\text{d} I$ is chosen as high as possible, and its order is chosen as low as possible. The resulting robustness filter Q_d is a 3^rd order Butterworth filter with a cut-off frequency of 28 Hz. The cut-off frequency and order are limited by the modeling errors of model $\hat{P}$ compared to true plant P, see Figure 6. The resulting optimal learning filter $L_{\mathcal{H}_\infty}$ is depicted in Figure 7. It can be observed that $L_{\mathcal{H}_\infty}$ closely matches $\widehat{SP}^{-1}$.

Second, a noncausal filter $L_{\mathcal{H}_\infty,comp}$ is designed based on the filter $Q = Q_\text{d} I$, where Q_d-filter is the exact same low-pass 4^th order Butterworth filter with a cut-off frequency of 23.5 Hz, as used in Section 7.1. This allows for a fair comparison of the suboptimal L_{ZPETC} with $L_{\mathcal{H}_\infty,comp}$.

8. SIMULATION EXAMPLE

In this section, the proposed approaches for multivariable ILC and multi-loop SISO ILC are analyzed by use of simulation on the system described in Section 6. The multivariable ILCs are designed in Section 7, the implemented multi-loop SISO ILC schemes are presented in Section 8.1. A fourth order reference signal $r = [r^x,\, r^\varphi]$ of $N = 2001$ samples is used, shown in Figure 8.

8.1 Design of Multi-Loop SISO ILC

Two multi-loop SISO ILCs, see Section 3, are designed for use in simulations. One multi-loop SISO ILC is designed by ignoring the interaction in the system, see Section 3.1. The Q-filter is designed according to (5), which does not guarantee monotonic convergence of the multivariable closed-loop system, and equals $Q = I$. The other multi-loop SISO ILC is designed by accounting for interaction through robustness, see Section 3.2. The filter $Q = Q_\text{d} I$ is designed according to (7), where Q_d is a low-pass 1^st order Butterworth filter with a cut-off frequency of 35 Hz. Both Q-filters are applied using the Matlab™ function `filtfilt` to get zero-phase behaviour, and both systems use equal L-filters as in (4), which form zero-phase error learning filters for $\widehat{SP}_\text{d}$, see Tomizuka (1987).

8.2 Simulation Results

Figure 9 depicts the matrix norm $\|e_j\|_2 = \bar{\sigma}(e_j)$, where $e_j = \left[e_j^x, e_j^\varphi\right] \in \mathbb{R}^{N\times 2}$, as a function of iterations. The time-domain error signals in trial $j = 12$ are shown in Figure 10. In the first trial $j = 0$, no feedforward is applied. The following observations are made:

- The multivariable ILC systems outperform the robust multi-loop SISO ILC system in terms of the asymptotic error. This is due to the design of the multivariable L-filters, which explicitly take interaction into account.

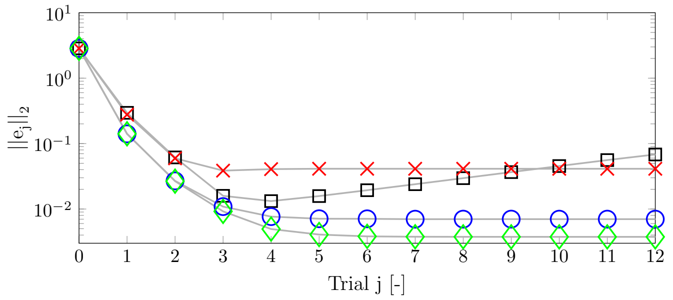


Fig. 9. The benefit of multivariable ILC design: i) the multivariable ILCs are explicitly designed for interaction (◇, ○), and outperform the multi-loop SISO ILC (×) which accounts for interaction through Q-filtering; ii) If the interaction is ignored, a nonconvergent ILC scheme (□) is obtained; iii) Through optimal $\mathcal{H}_\infty$ synthesis (◇) increased performance is achieved compared to the heuristic design (○).

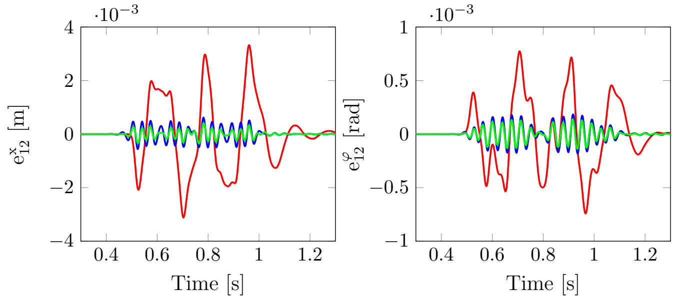


Fig. 10. The asymptotic error is significantly improved by the multivariable ILC designs ($\mathcal{H}_\infty$: —), (ZPETC: —) compared to the robust multi-loop SISO ILC (—).

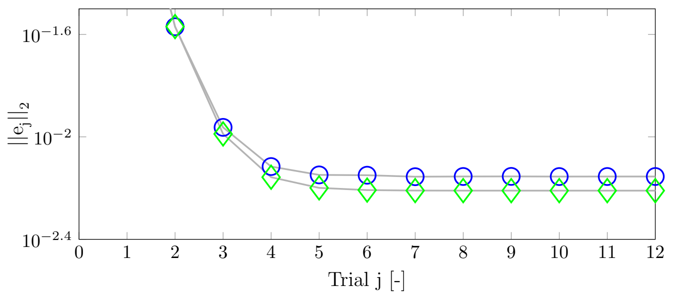


Fig. 11. Benefit of optimal L-filter design: the use of optimal $L_{\mathcal{H}_\infty,comp}$ (◇) improves performance compared to the heuristically designed L_{ZPETC} (○). Both approaches use the exact same filter Q.

Although the Q-filter of the robust multi-loop ILC has a higher cut-off frequency, this does not guarantee better asymptotic performance, see (3).

- The $\mathcal{H}_\infty$-optimal ILC design outperforms the suboptimal heuristic design, and yields a performance increase of 50% in terms of $\|e\|_2$.
- The error of the interaction-ignoring multi-loop SISO ILC system increases after four trials. This demonstrates that ignoring interaction in the design of an ILC can lead to a non-convergent closed-loop scheme.

In addition, the influence of the different design methodologies for multivariable filters L is compared. That is, the influence of L_{ZPETC} and $L_{\mathcal{H}_\infty,comp}$ on the achievable error, using the exact same filter Q. The results are shown in Figure 11, and the following observation is made:

- The use of $L_{\mathcal{H}_\infty,comp}$ improves the asymptotic performance by 12% in terms of $\|e\|_2$, compared to the heuristically designed L_{ZPETC}. Here, it is emphasized that both approaches use exactly the same Q-filter.

9. CONCLUSIONS

In this paper, frequency-domain ILC design techniques are analyzed and developed for MIMO systems. First, an analysis of multi-loop SISO ILC is provided. It is shown that performance of the ILC scheme may have to be sacrificed due to required robustness for neglected dynamics. Second, two design frameworks are proposed for multivariable ILC: i) an heuristic approach which extends ZPETC to MIMO systems, and ii) an $\mathcal{H}_\infty$-optimal approach with finite preview. Third, benefits of the proposed multivariable approach compared to multi-loop SISO ILC are demonstrated on an industrial printer model example.

Ongoing research focuses on further ILC design strategies for MIMO systems, further improving numerical accuracy of the proposed ZPETC algorithm, extending the ZMETC procedure by means of the proposed framework, and experimental validation on the industrial motion system.

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