Efficient Model-Free Iterative Learning Control for Massive MIMO Systems using Stochastic Approximation

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1 Background

Iterative learning control (ILC) aims to achieve high performance while providing robustness against model errors. Typical optimization-based ILC approaches use models combined with experimental data [1], and existing data-based methods such as [2] are experimentally expensive. This research aims to develop an efficient model-free ILC approach for massive MIMO systems using an unbiased gradient estimate that is obtained from a single experiment.

2 Problem formulation

Consider a MIMO system $J \in \mathbb{R}^{n_o N \times n_i N}$ with n_i inputs and n_o outputs, given in lifted form by

$$e = r - Jf \tag{1}$$

with input $f \in \mathbb{R}^{n_i N \times 1}$, error $e \in \mathbb{R}^{n_o N \times 1}$ and unknown exogenous disturbance $r \in \mathbb{R}^{n_o N \times 1}$. The criterion

$$\mathcal{J}(f) = \|e\|_{W_e}^2 + \|f\|_{W_f}^2 \tag{2}$$

with $\|x\|_W = \sqrt{x^\mathsf{T} W x}$ is minimized iteratively using a gradient descent algorithm with parameter update

$$f_{j+1} = f_j - \varepsilon_j g(f_j) \tag{3}$$

with step size ε_j and gradient

$$g(f_j) = -2J^\mathsf{T} W_e e_j + 2W_f f_j. \tag{4}$$

The gradient can be obtained using a model, or through $n_i \times n_o$ dedicated experiments on the adjoint of the system J [2, 3]. This research aims instead to use an approximation $\hat{g}(f_j)$ obtained from a single experiment.

3 Approach

A stochastic approximation adjoint ILC (SAAILC) approach is proposed, in which an unbiased approximation of the gradient is obtained from a single experiment as

$$\hat{g}(f_j) = -2\mathcal{T}^{n_i} A_j J A_j \mathcal{T}^{n_o} W_e e_j + 2W_f f_j. \tag{5}$$

The entries of matrix A_j are samples from a symmetric Bernoulli ± 1 distribution, and $\mathcal{T}$ is a time-reversal operator, for which it holds that $\mathcal{T} J^{11} \mathcal{T} = (J^{11})^\mathsf{T}$ for a SISO system J^{11}. Estimate (5) replaces $g(f_j)$ in (3), and since
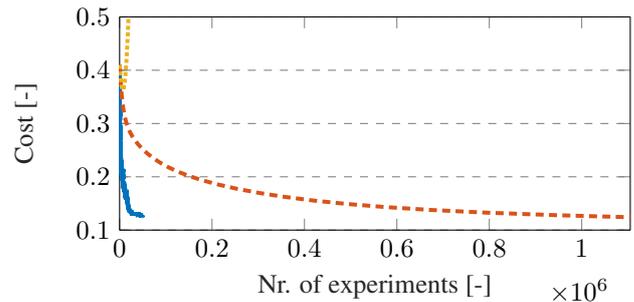

Figure 1: The cost as a function of the number of experiments in adjoint ILC for a non-symmetric 21×21 MIMO system. SAAILC (—) requires far fewer experiments to reach the same cost as non-symmetric deterministic adjoint ILC (- -), while symmetric deterministic adjoint ILC (·····) results in a diverging cost.

$\mathbb{E}(\hat{g}(f_j)) = g(f_j)$, the resulting algorithm can be interpreted as a Robbins-Monro type stochastic gradient descent algorithm, for which convergence can be shown.

The proposed SAAILC approach is experimentally advantageous compared to the deterministic approach in [2], which essentially applies the method by [3] to each subsystem of J subsequently, thus requiring $n_i \times n_o$ dedicated experiments to obtain the gradient for a non-symmetric n_i by n_o system.

4 Results

In Figure 1, SAAILC is illustrated using a random non-symmetric 21×21 MIMO system. It is shown that SAAILC achieves the same cost as the deterministic MIMO adjoint ILC algorithm, while reducing the number of required experiments significantly. In addition, it is shown that assuming that the system is symmetric in order to reduce the number of experiments results in a diverging cost.

References

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